# Predictions and Application of Queuing Analysis at Regional Hospital Limbe

Daphne Tatenda Machangara (daphne.machangara@aims-cameroon.org)
African Institute for Mathematical Sciences (AIMS)
Cameroon

Supervised by: Prof Habiboulaye AMADOU Boubacar
Universit´e de Lille, France
Supervised by: Prof Giovanni Andreatta
University of Padova, Italy

20 December 2018

*Submitted in Partial Fulfillment of a Co-op Masters Degree at AIMS-Cameroon*

# Abstract

This study concentrated on the application of queuing analysis and prediction of waiting time at Regional Hospital Limbe (RHL). The main purpose of the work was to be able to make mathematical sense of a real-life scenario concerning queues (waiting lines), and to come up with models for performance measure and improvement. This to be achieved using queuing theory concepts, composed of queuing models that provide some operational insights because of their analytical nature. The observations included studying patient arrival and waiting times, together with doctors' service times.

The results show the busy departments in the hospital, busy days and busy times. Long waiting times were found to exist mostly from both General Practitioner (GP) and specialist consultations. The queuing concept was applied to only one service segment, GP consultation. Although strong scientific conclusions cannot be made on the queuing models obtained because of inefficient data, the value of this work lies mainly in the methodology and proposal of different operating systems that can be adopted. Furthermore, some predictions were made using machine learning to see how long a patient can wait in queue for service, with the model predicting with an average of $10$ minutes $53$ seconds of error.

**Keywords**: Queuing theory, Queuing model, Queuing system, Arrival rate, Waiting time, Service rate.

## Declaration

I, the undersigned, hereby declare that the work contained in this essay is my original work, and that any work done by others or by myself previously has been acknowledged and referenced accordingly.

DAPHNE TATENDA MACHANGARA, 20 December 2018.

# List of Abbreviation

**RHL** Regional Hospital Limbe

**OPD** Out Patient Department

**GP** General Practitioner

**ICU** Intensive Care Unit

**DGOPH** Douala Gynaeco-Obstetric and Paediatric Hospital

**CT** Computed Tomography

**US** Ultrasonography

**FCFS** First Come First Serve

**MSE** Mean Squared Error

**FIFO** First In First Out

**LIFO** Last In First Out

**MAPE** Mean Absolute Percentage Error

**MAE** Mean Absolute Error

**RMSE** Root Mean Squared Error

**MedAE** Median Absolute Error

**SPSS** Statistical Package for Social Sciences

**ENT** Ear Nose Throat

**TAT** Turn Around Time

**CI** Confidence Interval

# Contents

# 1. Introduction

Almost on a daily basis, we are bound to wait for service, be it food outlets, banks, hospitals etc. In general, waiting is tiring, inconvenient by slowing productivity and brings about frustration. In hospitals, the longer patients wait, the more injuries suffered, which might even result in death. Increase in patient waiting time increases medical professionals workload and might reduce the quality of service. As a result, it is important to decrease patients' waiting time while improving their satisfaction. Through the application of queuing analysis, waiting time can be attempted to be solved by analysing the queuing system, developing a simulation model to help in deciding queue management and proposing other better models for use. The queuing methodology helps in deciding staff allocation needed to manage the services and the concept brings about an understanding of uncertain demand level for patients' service.

In this study, data was collected through questionnaires and observations. The mean number of arrivals, waiting and service times in selected departments were calculated, bringing about the busy days and times of the week. Using machine learning, predictions for both waiting and total time of a patient were made. The queuing concept was applied to model a queue in one selected department, give the queue model overview and determine its effectiveness measure.

Almost similar research has been done and published on queuing analysis in healthcare. Using and referencing some of these research papers, we attempt to further research, combine ideas and improve on queuing analysis in the RHL in particular.

## 1.1 Research questions

In this project, the main aim was to study the queues with the following research questions;

1. What is causing the queues?
2. Which are the busiest departments, busiest days and busy times of the day?
3. Is there a correlation between long waiting lines, day or time of the week?
4. How is the queuing system performing in terms of waiting and service times? Is it efficient enough?
5. Is it possible to have patient time predictions for the future?

All this is helpful in queuing analysis and attempting to reduce waiting time and come up with optimal decisions.

## 1.2 Definition of terms

The following definitions are in the context of a hospital or health facility centre.

Queuing theory: Queuing theory is the mathematical study of waiting lines, or queues [20].

Queuing model: A mathematical representation of the characteristics and limitations of the queue.

Queuing system: Any system where an arriving flow of patients request service and the servers in turn await to provide service to them.

Arrival rate: The number of patient arrivals per given unit of time.

Waiting time: It is the time interval a patient has to wait for service.

Service rate: The rate at which patients are being served in a system.

## 1.3 Study area: Regional Hospital Limbe (RHL)

This study was conducted in the period June - November $2018$ at RHL, Southwest Region of Cameroon. The population is approximated as being unlimited, that is an infinite number of patients come in on a daily basis. There are $200$ beds, an emergency department and the hospital is a principal referral for the region. In terms of medical personnel, there is $242$ active staff, of which $38$ are medical doctors, $28$ specialists and $10$ GPs. Of the $10$ GPs, $4$ are practising.

Three GPs work from $8$ a.m to $3$ p.m and hand over to another GP till $8$ a.m. Specialists are only available at most $4$ times a week and work only during the day. The laboratory is open $24$hrs although some tests, if not an emergency case, can only be performed during the day. For this study, records are for the day time only.

Whether a patient arrives and goes to the emergency room or Out Patient Department (OPD) first, they are likely to encounter many queues before they leave. Two types of patients exist, an old member (return patient), and one coming for the first time (new patient). The new member needs to pre-register on the information desk before they undergo any services. Patients arrive, wait for the service, obtain service and then depart as summarised in Figure 1.1.

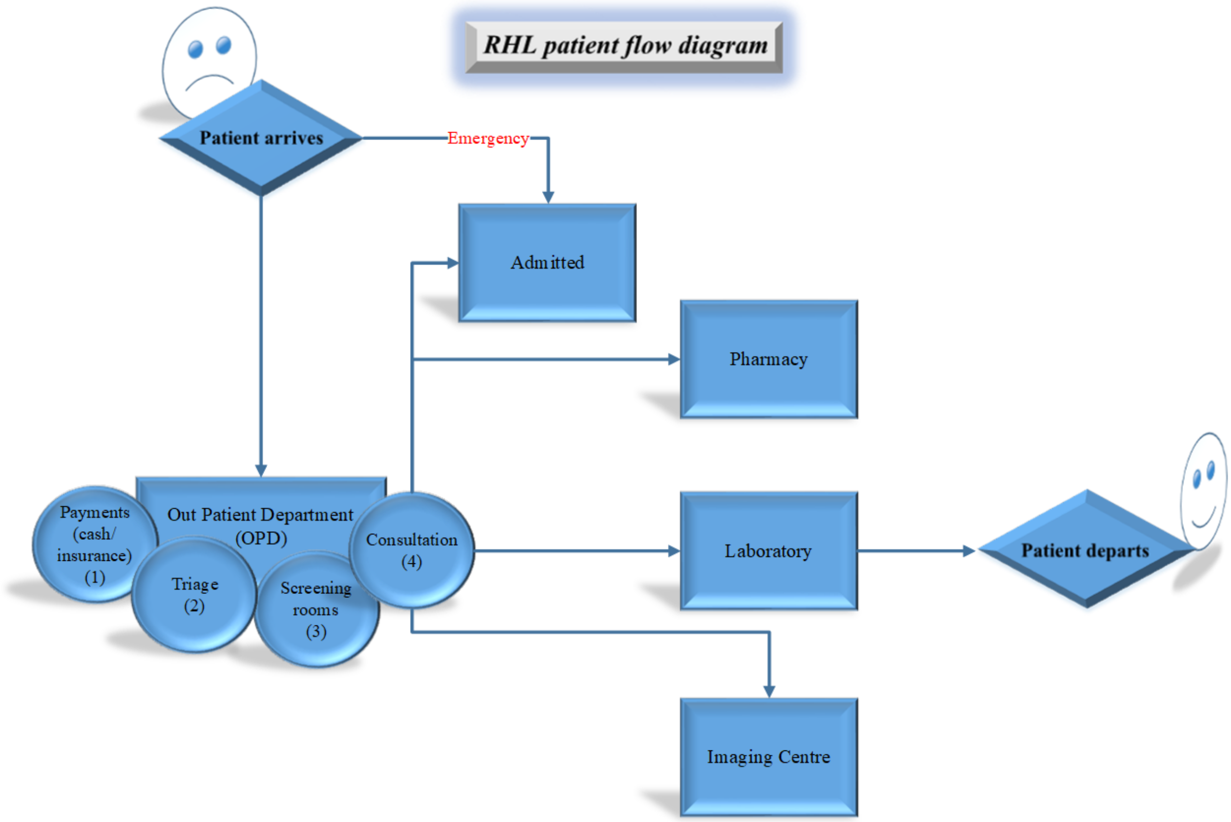



Figure 1.1: RHL patient flow illustration

In this study, service segment and department were used interchangeably, although OPD, Laboratory and Imaging centre are the departments and the service segments are the facilities found within them such as the group of specialists and so on.

# 2. Literature Review

## 2.1 Introduction

The aim of literature review is supporting the conducted research within the literature body and providing evidence for the reader using other published articles.

Waiting time has generally been a problem and many have tried to tackle that issue. Even British Columbia Medical Association [2] considers the most common problem in different departments of most healthcare organisations to be long waiting time. Acccording to the recommendation by the Institute of Medicine (IOM) [15], at least 90% of patients should be seen within 30 minutes of their scheduled appointment time. As to whether this is being implemented or not is still a challenge faced by many health facilities.

## 2.2 Theoretical review

**2.2.1 Queuing theory.** Queuing theory is a branch of Operations Research and it was first analyzed by A.K Erlang in $1913$ in the context of telephone facilities. Since then it has been applied to many different services.

Ozcan [16] describes the queuing theory as a mathematical approach to the analysis of performance parameters for queues in health care systems. Furthermore, according to Hall [10], the theory manages patient flow through the system, that is, if a patient flow is good, patients flow like a river, completing each stage with a minimal delay but when the system is broken, patients accumulate like a reservoir. In turn, queuing models have been widely used to model and analyse various health systems, such as hospitals [4].

**2.2.2 Data Science and machine learning.** Data science is more about the extraction of information from data to answer particular questions or solve particular problems. Machine learning, which is a part of data science, uses algorithms which learn (predict) models from data.

## 2.3 Previous studies

Waiting time differs not only in countries but even in centres within those countries. It is a major problem for both developed and developing countries and some reports on long waiting times have been made, although most date $20$ years back.

From the studies carried out in developing countries, Fouogue [9] did a study in Cameroon at Douala Gynaeco-Obstetric and Paediatric Hospital (DGOPH) on patients who underwent laparoscopic surgery in the period November $2015$ to July $2016$. Although he did not go deeper in waiting time research, the study showed that the highest number of complaints concerning waiting time dominated by $73,3\%$.

In Nigeria's health centre for Ado-Ekiti University, Adeleke [1] considered the waiting of patients as a single-channel queuing system with Poisson arrivals and exponential service rate on First Come First Serve (FCFS) with an $\mathrm{m/m/1}$ queuing system. It was discovered that on total, patients wait for $945$ minutes with a traffic intensity of $84\%$ (the probability of patients queuing on arrival), clearly showing that the service in the health centre is not $100\%$ efficient. Still in Nigeria, in the city of Benin, Dansky [7] found an average waiting time of about $173$ minutes.

Furthermore, Conrad [5] carried a study in Mulago hospital Uganda in Kampala city. The study showed long waiting times existing in $3$ departments, mainly the pharmacy ($123$ minutes), X-ray ($105$ minutes) and registration ($66$ minutes). He performed multivariate analysis and found that the variables associated with waiting time were time of arrival, day of arrival and number of patients in the queue.

Wafula [18] carried a project in the University of Nairobi health Services senior staff clinic (S.S.C). She showed that on average patients spent $55.3$ mins at different service points in the clinic with the longest average waiting time of $13.1$ minutes being at the doctor's area. Patient arrivals were noted to be higher in the morning compared to any time of the day by $47.9\%$.

In the USA, an average waiting time of about $60$ minutes and $188$ minutes was found in Atlanta and Michigan respectively by Stewart [19], while in University College Hospital Ibadan, Bamgboye [3] observed a mean waiting time of $73$ minutes.

Moreover, in an Indonesian public hospital, Mardiah [13] observed $11$ specialists department and noted the busy days to be Monday and Tuesday, with a large number of patients coming between $8$ and $11$ a.m at an arrival rate of $30$/hr. The waiting time ranged between $27 - 51$ minutes. Of the specialists, the most frequented one was internal medicine.

In terms of modelling hospital settings by applying queuing theory, a recent analysis was performed in an eye hospital (Dr Yap Eye Hospital Yogyakarta) in Indonesia using the queue system by Putri [17]. Based on observations, the model was of type (M/M/$1$):(GD/$\infty$/$\infty$) with a non-steady-state condition and $3$ methods were used, among which was Monte Carlo Simulation. The results of interest from Monte Carlo Simulation showed the effectiveness measure of the average waiting time to be $36.65$ minutes and a queue of $11$ patients long on average.

On the other hand, Kim [12] makes use of queuing analysis and simulation to enhance performance in the Intensive Care Unit (ICU) department of a public hospital in Hong Kong. The results show an efficiency on the bed utilization rate by $69\%$, with a patient spending approximately $7$ hours before gaining actual entry, and on average $3$ days in the system.

Among the researchers in a radiology department, Vasanawala [22] was to determine whether queuing theory would allow prediction of an optimal number of schedule slots to be reserved for urgent Computed Tomography (CT) and Ultrasonography (US). However from findings, in the case of CT, the model could not be used to perfectly predict demand.

Also, Fomundam [8] seeks to show the applicability of queuing theory and their analytical models, from the perspective of healthcare organizations, where waiting time, and utilization analysis, system design, and appointment systems are summarised results.

Concerning predictions, Curtis [6] carried out a study in the Massachusetts General Hospital Department of Radiology between July $2016$ and January $2017$ for application of machine learning models to predict waiting times. Different machine-learning algorithms such as neural network, random forest, support vector machine, elastic net, multivariate adaptive regression splines, k-$^{th}$ nearest neighbour, gradient boosting machine, bagging, classification and regression tree, and linear regression were used to find the most accurate method. Among all, the elastic net model performed best for predicting waiting times across all four modalities.

In addition, Troccoli [21] applied machine learning to predict the time a student in the Stanford University, department of Computer Science will wait before being helped, and the service time taken by the staff to resolve that student's problem. The prediction task was framed both as a problem of regression (applying neural networks) and a problem of classification (applying logistic regression and neural networks). For classification, neural networks and logistic regression were very comparable; for regression, a neural network was able to predict waiting times and helping (service) times with a Mean Squared Error (MSE) of $1756$ and $387$ respectively. Overall, the best results were from classification methods, accurately classifying up to $42\%$ of waiting times and $18\%$ of helping times within $5$-minute buckets, and up to $73\%$ of waiting times and $59\%$ of helping times within $20$-minute buckets.

However, above it all, Mcquarrie [14] shows that to minimise waiting times, the possibility is by giving priority to clients who require shorter service times.

# 3. Methodology

This chapter sets out our project's methodological framework. It describes the methods and instruments used to collect the necessary information and the procedure for analysing and interpreting the collected information.

## 3.1 Data collection

Instruments used in data collection were questionnaires and observations. Registers with required information from a few selected departments were also used.

**3.1.1 Questionnaire survey report.** Questionnaires (in French and English) were distributed to both patients and medical personnel in order to get opinions, time intervals and ratings and an overall insight on the functioning of the hospital. The questionnaire samples are given in Figure A.1 and Figure A.2.

**Background to survey**

The survey was conducted by the researcher in a time frame of $28$ days, using convenience sampling method. The patient response rate was $84\%$, with a total of $170$ respondents and $32$ refusal cases. Medical personnel had a $100\%$ response rate with $26$ respondents and no refusal cases.

For research simplicity, the OPD department mentioned in the questionnaire composed of $4$ service segments,

1. almoner - (cash payments),
2. triage - (parameter observations),
3. screening room - (registration of patient details, complaints and taking of vital parameters),
4. consultation (both GP and specialist doctors).

**3.1.2 Observation report.** Since perceptions can be exaggerated and biased, the study had to be complimented by observations which were done on arrival patients in respective departments for their various services. A stopwatch was used to time and record such important variables as service time taken, waiting time and patient arrival rate. In total, the number of days taken for observations was $43$. Initially, $3$ departments were to be observed, namely OPD, laboratory and imaging centre, but because of the complexity of the imaging centre, it was omitted. The OPD has $4$ service segments but only $2$ were selected, that is, the almoner and consultation (GP and specialists). From the specialists, only $3$ were observed, the Ear Nose Throat (ENT) ($2$ servers alternating on different days), Internist (only $1$ server) and cardiologist ($2$ servers working in parallel). The reason for not considering other specialists was because either they had a finite population which was not part of the study or they rarely had queues.

**3.1.3 Data analysis.** The questionnaire and observations data generated was entered in Statistical Package for Social Sciences (SPSS) version $20$ and analysed using Python. The analysis was in terms of general descriptions and visualisations, as well as performance of correlation tests on some observed variables.

## 3.2 Research Design

Two methodologies were used in this research, Machine learning for predictions and Queuing theory.

## 3.3 Machine learning

Machine learning is the study of computer algorithms that can learn to perform a task on the basis of their own experience. Under machine learning, there are several learning algorithms used for prediction, but this study focused on random forest only.

**3.3.1 Random forest.** This study made use of random forest. It is flexible, easy to use and it is the most used among other algorithms not only because of its simplicity but also because it can be used for both regression and classification. Random forest is a supervised machine learning algorithm that creates a forest and makes it random. The way random forest works is by building and merging several decision trees in order to obtain a more accurate and stable forecast. It adds randomness to the model while the trees grow. It searches for the best feature among a random subset of features instead of searching for the most important feature while splitting a node. As a result, there is a wide variety that usually leads to a better model. Therefore, only a random subset of the features is taken into account by the algorithm to split a node in the random forest. One can even randomise trees by using random thresholds for each function instead of looking for the best possible thresholds (like a normal decision tree). As much as the random forest has its advantages and disadvantages, one of the important points is its ability to provide a pretty good indicator of the importance it assigns to ones features.

**3.3.2 Prediction with machine learning.** Predictions for both waiting and total time spent were made using the Jupyter notebook in python. The main objective was to check the possibility of predicting the waiting time and total time a patient spends in the hospital on any other day and by how much accurate that model is. The problem was a supervised regression machine learning problem since both hospital data and times to be predicted were available and also real values. This type of machine learning requires lots of data and the model is able to be trained as well. The observed data was split into $\frac{2}{3}$ train data and $\frac{1}{3}$ test data, fitting the model using the Random Forest Regressor. Initially, the model had $1000$ decision trees which were further trimmed for simplicity to only $3$ levels with $10$ decision trees. For measuring prediction accuracy/error size, several measures exist like Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE).

**3.3.3 MAE and MAPE.** In our study, only the MAE and MAPE were used to find the error size. Although MAE was more favourable than MAPE in terms of describing the average error and how big of an error we can expect from the forecast.

Another important step was to use feature importance to find out the most important or relevant variables for predicting these times.

## 3.4 Queuing theory concept

Queuing theory has already been defined and described in previous chapters. In this section, we will illustrate the queuing system and their various components. Formation of queues has two important properties, maximum size (capacity of a system)/queuing capacity and queuing discipline. The size is the population which can either be limited (finite) or unlimited (infinite). Queuing rules for selecting patients for services are called queue discipline and they can be classified as First In First Out (FIFO)/ Last In First Out (LIFO)/Priority and so on. For simple queues, some queuing theory formulas exist, while complex situations will need computer simulation.

Two basic approaches are available for analysing queue systems.

1. *Analytical approach:* The method has an advantage on simplest models by providing summary measures which are simple to interpret. It attempts to find formulas (some algorithms) for calculating steady-state performance measures of the system.

2. *Simulation approach:* This approach has two types, simulating a known distribution and simulating a non specified distribution (bootstrapping). The method itself has more flexibility in comparison with the analytical approach. It simulates random elements of the system at the same time keeping track of the events as they occur through time. It builds a simulation model that is a computer model that mimics the situation in real life. This model is similar to other mathematical models but incorporates uncertainty in one or more input variables. The benefit of computer simulation is the ability to answer what-if questions without actually changing the physical system.

**3.4.1 Basic queue.** We begin by illustrating a simple queue as shown in Fig 3.1, where queuing behaviours like reneging and baulking are exhibited.

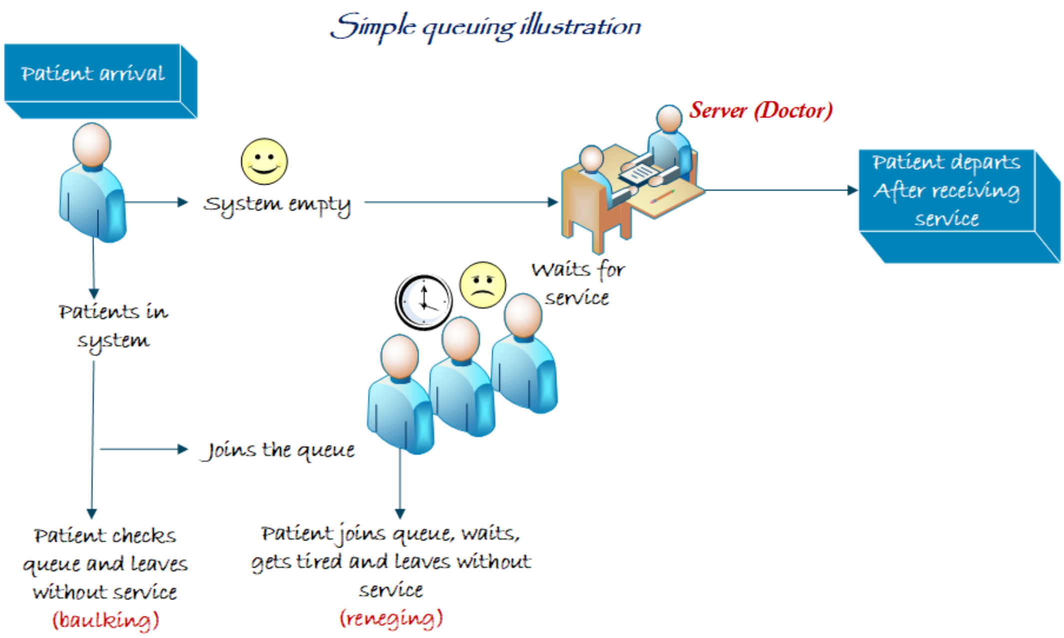



Figure 3.1: Illustration of a simple queue

The illustration simply shows the level of patience from when a patient enters the system up to when they are either served or they decide to give up and leave.

**3.4.2 Kendall's notation.** Next, we talk about the queuing concept model known as 'Kendall's notation', in its simplest form A/B/C, which was formalised by David George Kendall [11]. He was an English mathematician and statistician known for probability, statistical shape analysis and queuing theory and his notation is used in the queuing system description. The explanation is given by Fig 3.2.

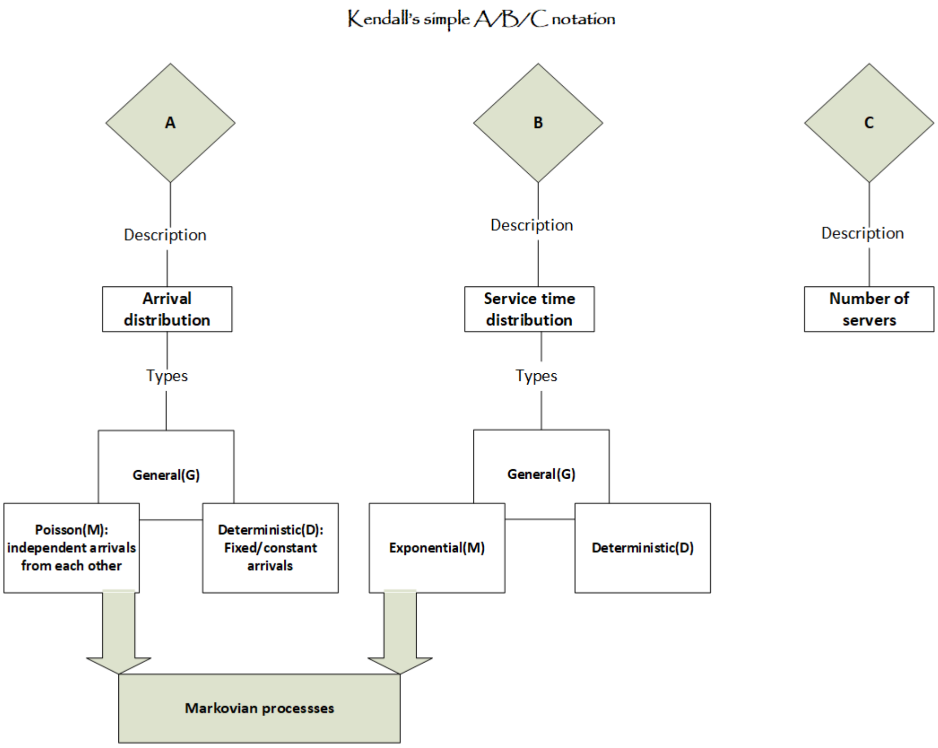



Figure 3.2: Kendall's notation

**3.4.3 Queuing theory assumptions.** The extension of 'Kendall's notation' is given by A/B/C/D/E, where D is the queuing capacity and E is the queuing discipline. By the basic assumptions of queuing theory, using the 'Kendall's notation', the following apply;

1. A: Is assumed to follow a poisson distribution and is a Markovian process.
2. B: Is assumed to follow an exponential distribution and is a Markovian process.
3. C: Is a positive integer.
4. D: Is assumed to be $\infty$, implying that noone is turned away when they come for service.
5. E: Is assumed to be FIFO/FCFS.

The basic model notation is thus given as M/M/C/$\infty$/FIFO/FCFS, where 'M' is the notation for Markovian processes. The Markov process can be defined as a random process where the future, given the present, is independent of the past and assumes the arrival or service rate. The exponential distribution exhibits an important property of being memoryless, that is, time for the next arrival is independent of when the last arrival occurred. Its characteristics include equal mean and standard deviation.

If these assumptions are met, then the analytical approach can be used, which makes use of Little's formulas, with the most important formulas given as follows,

$$W_q = \frac{\rho}{\mu(1-\rho)} \quad \text{(Average waiting time in the queue).} \tag{3.4.1}$$

$$W_s = \frac{1}{\mu(1-\rho)} \quad \text{(Average waiting time in the system).} \tag{3.4.2}$$

$$L_q = \lambda W_q \quad \text{(Average number of patients in the queue).} \tag{3.4.3}$$

$$L_s = \lambda W_s \quad \text{(Average number of patients in the system),} \tag{3.4.4}$$

$$\text{where} \quad \rho = \frac{\lambda}{\mu} \quad \text{given that} \quad \lambda < \mu, \quad \text{for} \quad \lambda, \mu > 0.$$

$\rho$: is the traffic intensity and it measures the average occupancy of a server. If $\rho > 1$, it implies the queue grows without bound, therefore to be able to control the queue, $\rho < 1$.

$\lambda$: is the average arrival rate of patients.

$\mu$: is the average service rate of patients.

The notation for a poisson distribution is given as $X \sim$ Po($\lambda$), with $\lambda$ as the only parameter. On the other hand, the notation for an exponential distribution is given as $X \sim$ Exp($\mu$), where $\mu$ is the only parameter.

For many queuing situations, arrivals occur randomly, and the occurrence of the next arrival cannot be predicted. Among the distributions representing the time between successive arrivals, the most important is exponential distribution.

In terms of hospital settings, most arrivals are modelled by a poisson distribution, where patients arrive independently one after another and an exponential service distribution. An example of deterministic arrivals in a hospital setup is when consultation is by appointment and patients have to come at a fixed given time. A general distribution is non specific and could be any kind of distribution.

**3.4.4 Types of queues.** Queues may be organised in different ways and their various types are shown in Fig 3.3, where only $3$ common types have been illustrated. The term server refers to a doctor.

In some systems, patients have a choice to choose the first free doctor or they just maintain the queue to a fixed doctor they have been assigned to. The patients are then selected for service by the various queue disciplines.

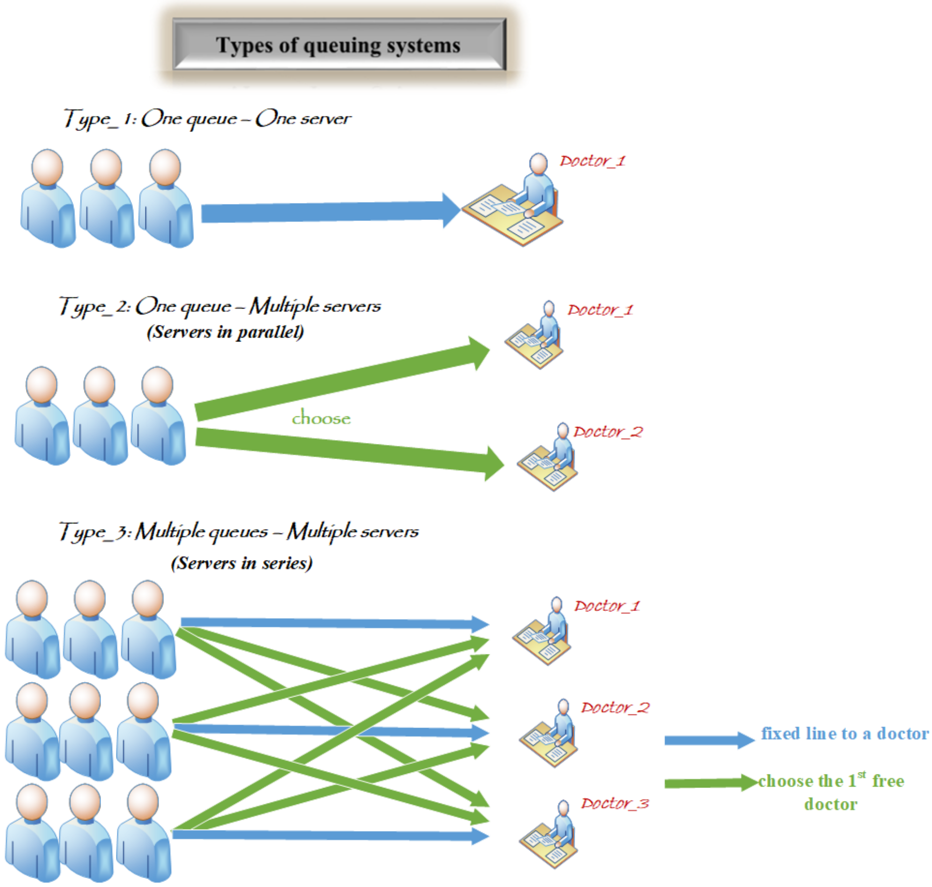



Figure 3.3: Types of queues

### Queuing theory Case study

The only department chosen was the consultation by GP mainly because of time constraint and lack of workforce. The system belongs to *Type_3* of Fig 3.3, with 3 consultation rooms, implying 3 servers, with patients having a fixed queue, although at times if a doctor gets free they can select patients from another queue, (which makes the situation complex). The patients were coming from an infinite population meaning the system was able to receive all the patients coming for consultation. The queuing discipline was FIFO, although at times priority and emergency cases were treated first, making the system preemptive.

During observations, the system was noted to be more complex than anticipated and the queuing theory assumptions were violated. For example, by assumption of healthcare waiting lines, the system service time was supposed to exhibit exponential distribution properties but it did not since there was no continuity in serving of patients. The arrival process was not constant over time as well and it could not fit the poisson process. Hence the arrivals and service time were instead categorised as a general distribution, meaning it could be any kind of a distribution. As a result therefore, the assumption of modelling it as an M/M/1 and using analytical approach was ruled out. The system was observed to be a $3\times$ G/G/$1/\infty$/FIFO or priority.

The observations were on 4 different days, but 3 of them were unsuccessful because of several disturbances. One of the unsuccessful attempted observation is shown in Figure A.4. In the end, only observed data shown in Figure A.3 was used. The 3 observed doctors, named 'Doctor_1', 'Doctor_2', 'Doctor_3', had 19, 12 and 13 patients observed respectively to have arrived in the period $7:30-12:40$. The most important parameters that were taken note of were the arrival time, waiting time, service time per hour and of cause number of servers.

**3.4.5 Simulation process.** Since the system was a bit complex, only the simulation approach (to find averages) on this system was possible, instead of using the standard queuing theory with its associated mathematical formulas. The type of simulation used was bootstrap simulation since the service time was not from a specified distribution.

The tool for this analysis was Excel, where the observations were used to perform a simulation. The initial step for this approach involved reforming probability distribution based on random number generation in reference to the real data, where the random numbers were generated by the function RAND(). One simulation was performed per doctor for each of the 3 using the VLOOKUP() function, to simulate 1000 replications, where each replication is an independent replay of the occurring events. The replications were generated using a data table. To do this, the observed data in the spreadsheet was used to construct a typical "prototype" of the simulation. Also, a 95% Confidence Interval (CI) was estimated. Part of the replication process table which was done for 'Doctor_1' is shown in Figure 3.4.

| No. Replications | Average time in system | Average time in queue | % time server is idle |
|---|---|---|---|
| | 22.4 | 13.72 | 0.1033058 |
| 1 | 15.32 | 8.72 | 0.5657895 |
| 2 | 8.8 | 2.52 | 0.6884921 |
| 3 | 8.68 | 2.08 | 0.4728435 |
| 4 | 9.52 | 2.16 | 0.5523114 |
| 5 | 12.8 | 5.92 | 0.4723926 |
| ⋮ | | | ⋮ |
| 995 | 14.24 | 6.76 | 0.5630841 |
| 996 | 11.2 | 3.84 | 0.54 |
| 997 | 18.12 | 10.4 | 0.4373178 |
| 998 | 11.12 | 4.72 | 0.638009 |
| 999 | 8.84 | 2.08 | 0.5517241 |
| 1000 | 9.88 | 2.16 | 0.6553571 |

Figure 3.4: Replication process for 'Doctor_1'

The simulation analysis was also useful to investigate what would have happened if a different policy or strategy was used. After observations, the simulation model was used to compare 2 situations. The first was the observed system "with disruptions", where the doctors during consultation had other things to attend to. The other situation was termed a system "without disruptions", for comparison, where the doctors consult only without any other disturbances.

One of the virtues of the simulation is that it allows us to experiment with alternatives. Although not used in this study, other alternatives could include simulating a system where the number of doctors is increased to see the difference that can be made.

## 3.5 Challenges and limitations of the project

Each research comes with its own challenges and setbacks. During this project, the following were faced,

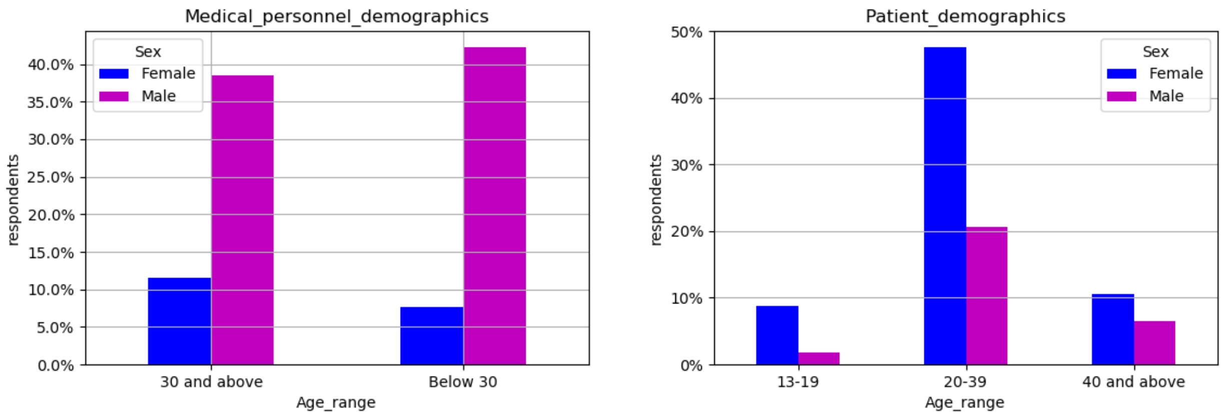
1. Monday ghost town: Since the beginning of June the existence of ghost town on Mondays made it impossible to observe or record any data on Mondays. Therefore observations were done Tuesday to Friday.
2. Questionnaire administration:
   - For the Francophone, it was sometimes difficult to converse.
   - Unwillingness for some patients to participate in answering the questionnaires for either health or personal reasons.
3. Study samples: Overall study samples were not as large as desired.
4. Lack of workforce: Carrying out observations by one person (me) was difficult, hence department observations had to be alternated, causing some bias as there was unequal sharing of time.
5. Application of queuing theory concept: The service segment chosen was the GP consultation and the system was a bit complex than perceived in the following sense,
   - Doctors unavailability: There was non-uniformity of doctor arrivals and departure, as a result doctor availability was inconsistent.
   - Data size: As a result of the complexity, the data was not enough to draw strong scientific conclusions, only analysis and performance measure could be depicted.
   - Mixture of queuing discipline: The queuing discipline was not only FIFO, as at times emergency, priority cases or even random selection of patients would result and disrupt the whole observation.
   - Different types of queuing systems: The model was supposed to be $3$ lines each leading to one of the $3$ doctors, but it was not always the case, as sometimes $2$ doctors shared $1$ line.

# 4. Result analysis and Discussions

In this chapter we focus on summarised results and discussions from the study conducted.

## 4.1 Questionnaire Descriptives

**4.1.1 Demographics.** We begin by the demographics from both patient and medical personnel questionnaire respondents in relation to sex and age as shown in Figure 4.1.




(a) Medical personnel respondents (b) Patient respondents

Figure 4.1: Questionnaire demographics

The majority were male medical personnel below the age of $30$ and female patients of age range $20-39$.

Responses from various categories were analysed. Some of the questions were similar for both medical personnel and patients.

**4.1.2 Perception of departments with longest waiting times.** Both medical personnel and patients responded with the same top $3$ departments with long waiting times. In order of longest waiting time, OPD, Laboratory and Imaging centre were the departments said to have the longest waiting time in the hospital.

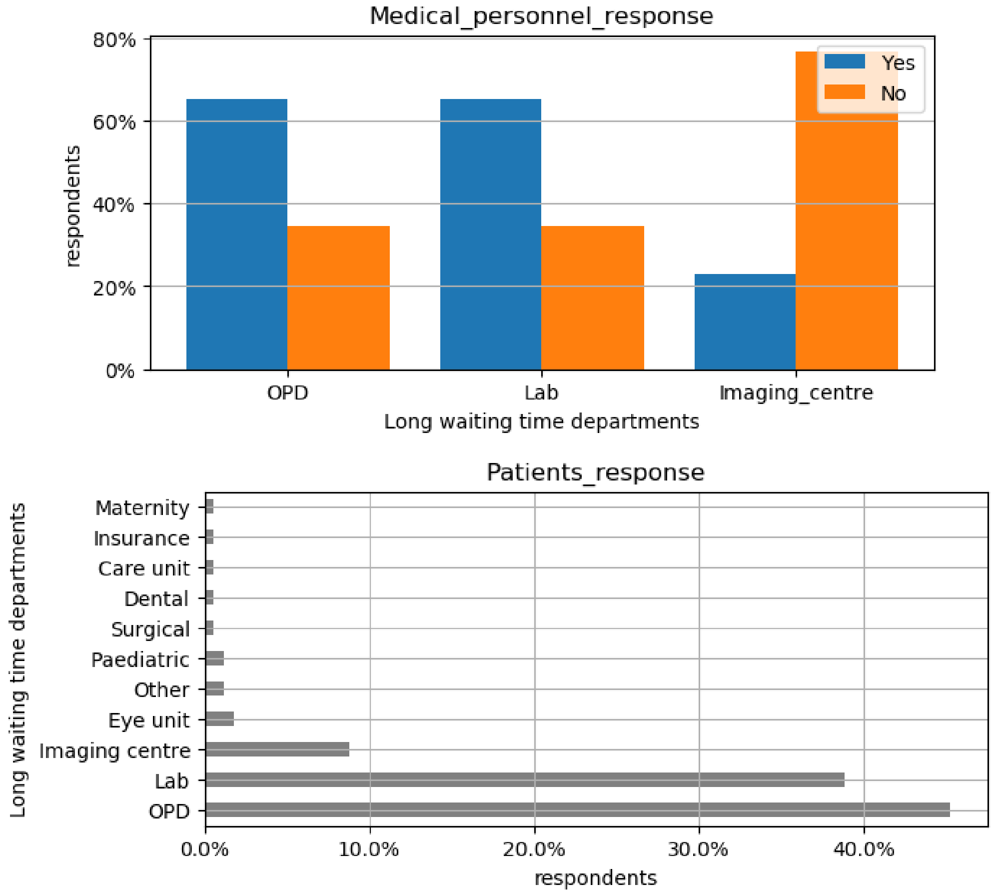



Figure 4.2: Perception of departments with long waiting times

**4.1.3 Waiting and service time approximations.** Patients responded to the approximate times they spent for waiting and receiving service. The OPD and laboratory had the same longest waiting and service times, both at more than $1$ hour and $10 - 20$ mins respectively. The imaging centre had both waiting and service times approximated at $30$ mins $-1$ hr as shown in Figure 4.3.

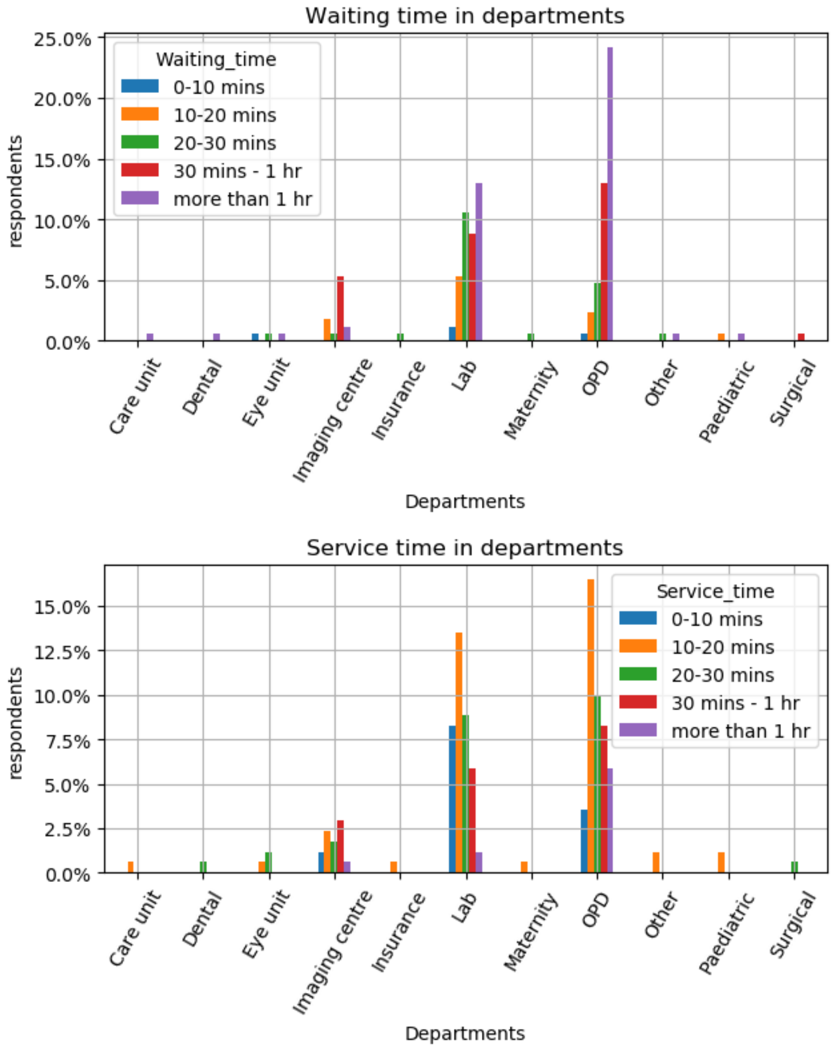



Figure 4.3: Perception of waiting and service times in departments

**4.1.4 Patient satisfaction on time spent in the hospital.** Another question for patients was to rate how satisfied they are concerning the amount of time they spent in the hospital and their responses were recorded in Figure 4.4, where the satisfaction rate had a scale of $0$ to $10$.

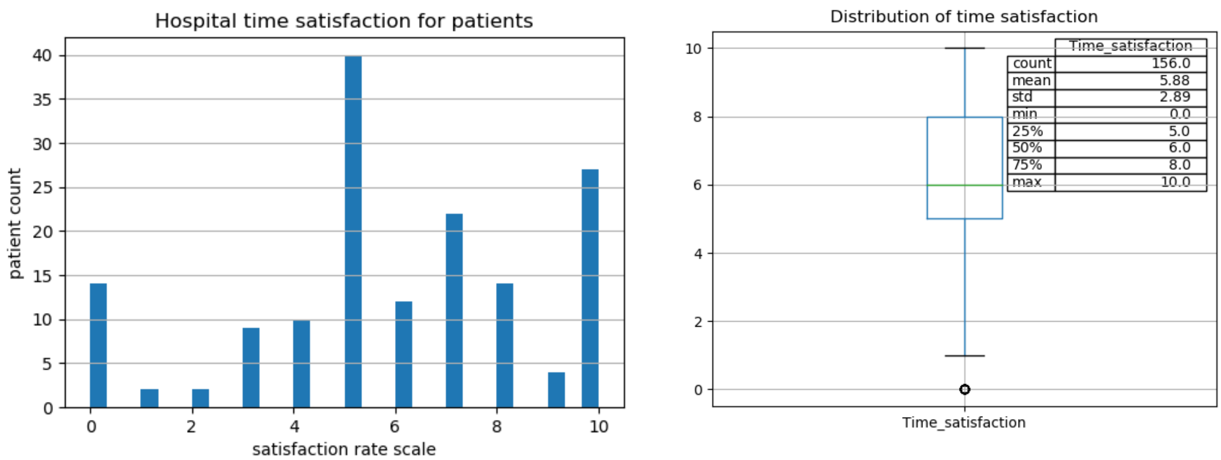



Figure 4.4: Patient time satisfaction ratings

We observe the impact waiting time has on patients' satisfaction and we can note that the modal satisfaction had a score $\frac{5}{10}$.

For interest sake, a satisfaction distribution comparing sex and age range was constructed in Figure 4.5 where the average satisfaction is indicated in green. We notice that males are more satisfied compared to females and the age range $40$ and above is more satisfied than other age groups. From conversing with the patients, the elderly seem to be more satisfied because they say they have no option, what matters to them is having specialists who can attend and treat them.

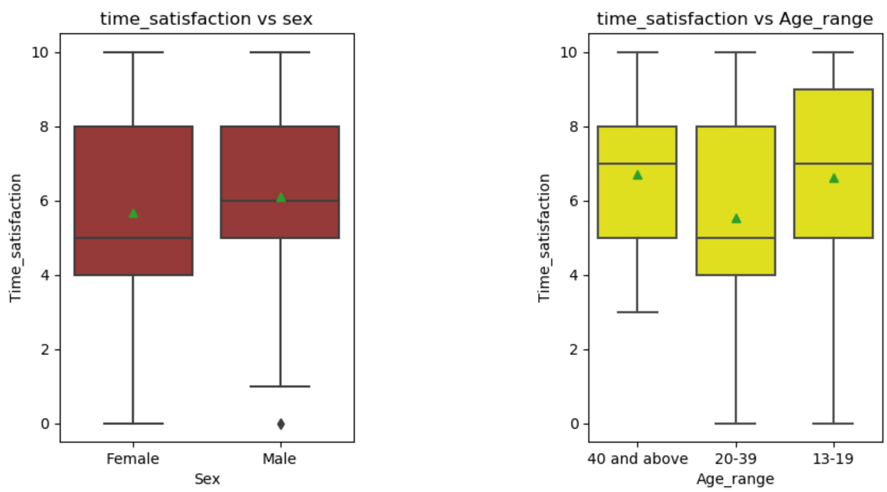



Figure 4.5: Patient time satisfaction comparison

**4.1.5 Busy days and times.** Medical personnel responded to the busiest days and times in the hospital. Monday was found to be the busiest and $96\%$ responded to mornings being busy of all times. The term busy being associated with a high number of patient arrivals and long queues.

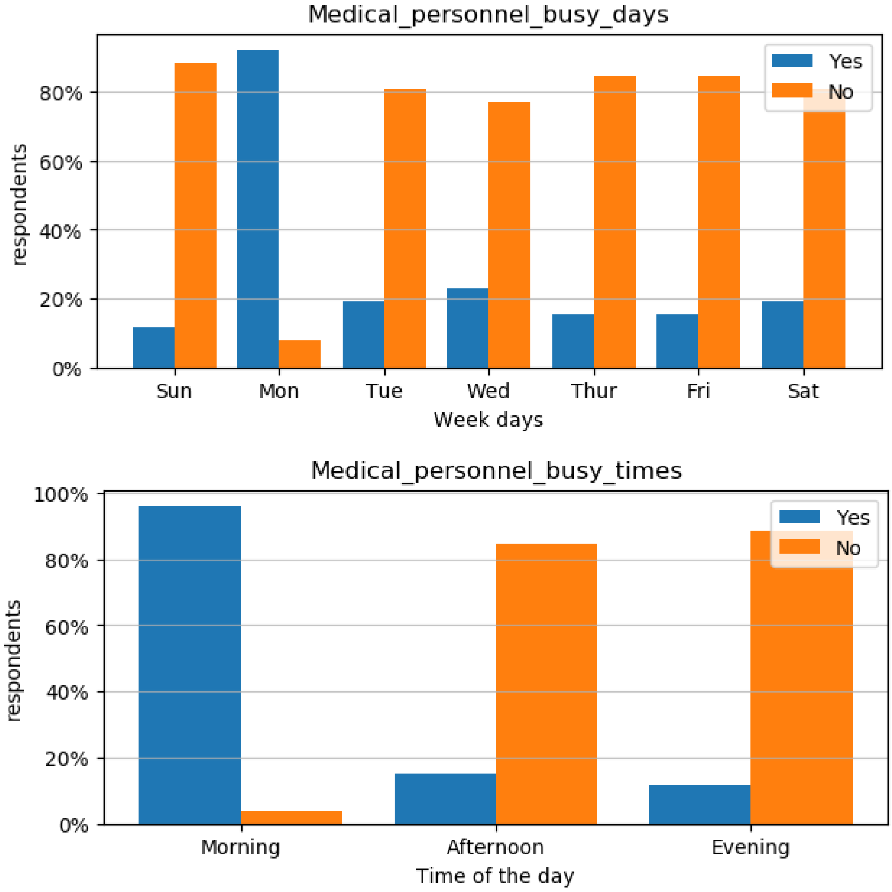



Figure 4.6: Perception of hospital busy days and times

**4.1.6 Perception of queue length.** Medical personnel responded and rated on how they viewed the length of queues generally in various departments. As shown in Figure 4.7, $57.7\%$ responded on the queues being generally average.

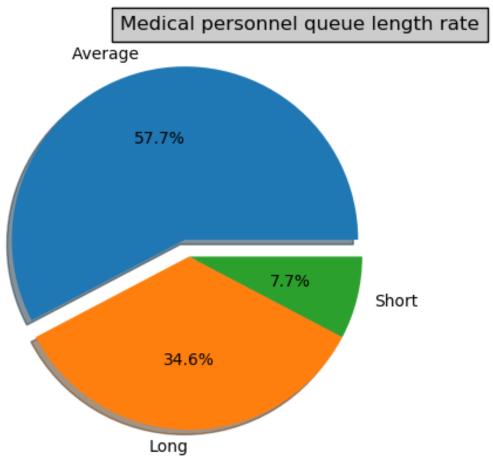



Figure 4.7: Perception of queue length

**4.1.7 Comments.** Both patients and medical personnel had a comment section to fill out, although the majority did not respond, the captured comments were just to give an insight on raised concerns in Table 4.1. Some of the comments from patients contradict each other but we notice a similarity in most comments from both the patients and medical personnel. We can also note that there was only 1 positive feedback comment.

Table 4.1: Comments

| | Patients comments | Medical personnel comments |
|---|---|---|
| Positive feedback | Satisfied with services | – |
| Negative feedback | Reduce waiting time<br>Doctors should be time conscious<br>Employ more medical personnel<br>Satisfied with services<br>Increase specialists<br>Lab results take time<br>Treat emergencies first<br>Give patients full attention<br>Lack of seriousness<br>Unsatisfactory services<br>Doctors not always available<br>Staff need a follow up<br>Give priority to aged people<br>Toilets in bad state<br>Polite and caring nurses<br>There is need for improvement<br>Increase lab equipment<br>Equal treatment when queuing<br>Rude nurses<br>Too much population<br>Inform patients if doctors will delay<br>Arrange benches properly | Long waiting time for patients<br>Staff lacks duty consciousness<br>Increase medical personnel<br>Remove unnecessary payment steps<br>Improve patient reception<br>Increase lab sample points<br>Attend patients immediately<br>Review patient-staff communication<br>Review the time results take to come out<br>Imaging centre results take too long<br>Sometimes doctors are unavailable |

## 4.2 Observations: Patient arrivals

After the feedback from the questionnaires, the top $3$ perceived departments with long waiting times were the ones used for observations, with the exception of the imaging centre. Therefore the observations narrowed to $2$ departments with $6$ service segments which were represented as follows,

1. OPD
   (i) Payments
   (ii) GP
   (iii) Internist
   (iv) ENT
   (v) Cardiologist
2. Laboratory

To understand patient flow, the number of patients arriving each day and monthly was taken note of and analysed. This information was also useful to understand if there was a link between number of arrivals and days of the week.

**4.2.1 Daily and monthly arrivals.** Screening room registers were used to obtain information for both daily and monthly arrivals. For daily arrivals, a sample of $4$ months (January, February, March and April $2018$) with $120$ days was randomly taken, and an average daily arrival of patients was calculated from Sunday to Saturday. On average, $53$ patients come daily (exception of appointment patients), with Monday, Wednesday and Tuesday being the top $3$ days with most patient arrivals in order. From the distribution plot, the maximum number of arrivals shown occurred on a Monday whereas the minimum arrivals occurred on a Sunday.

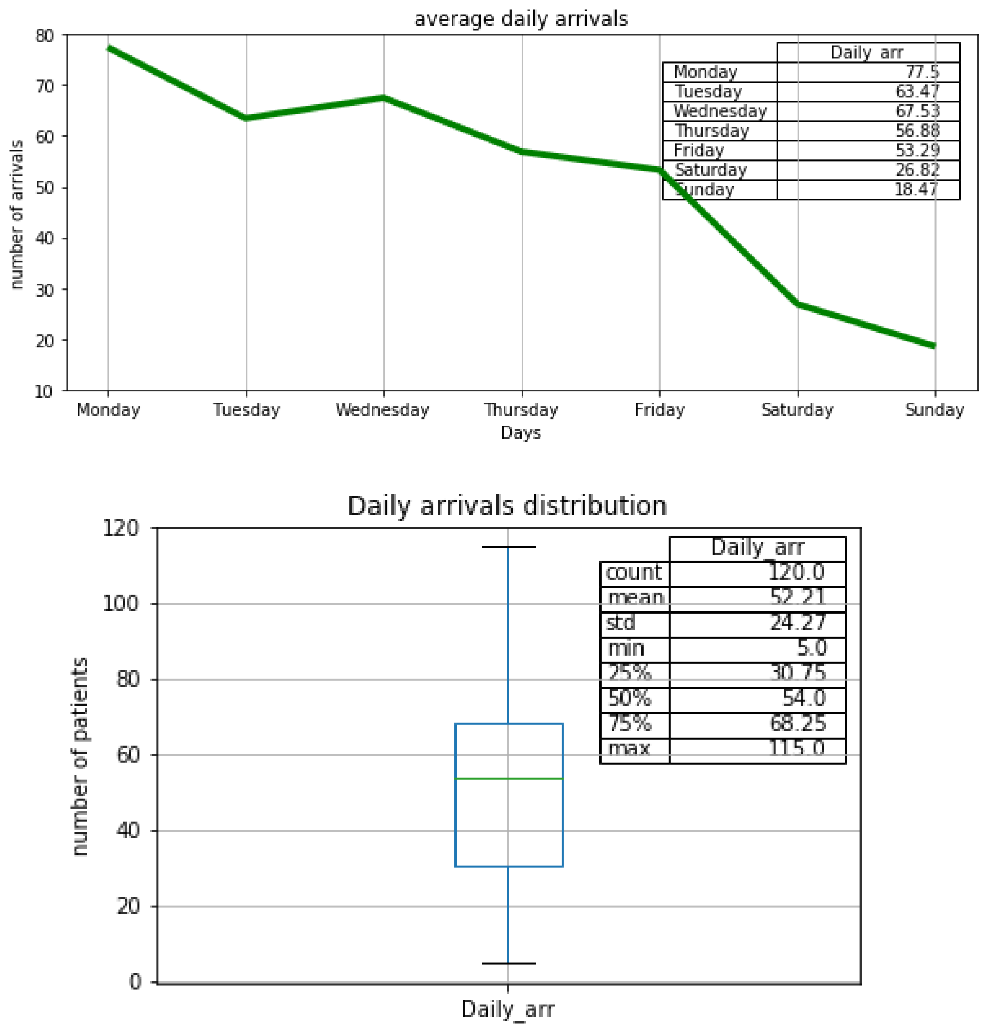



Figure 4.8: Daily patient arrivals

Monthly arrivals are presented in Figure 4.9 where only the first $6$ months of $2018$, January-June were used, with an average of $1533$ patients coming monthly. We notice a decrease in the number of patients monthly and it is a cause for concern.

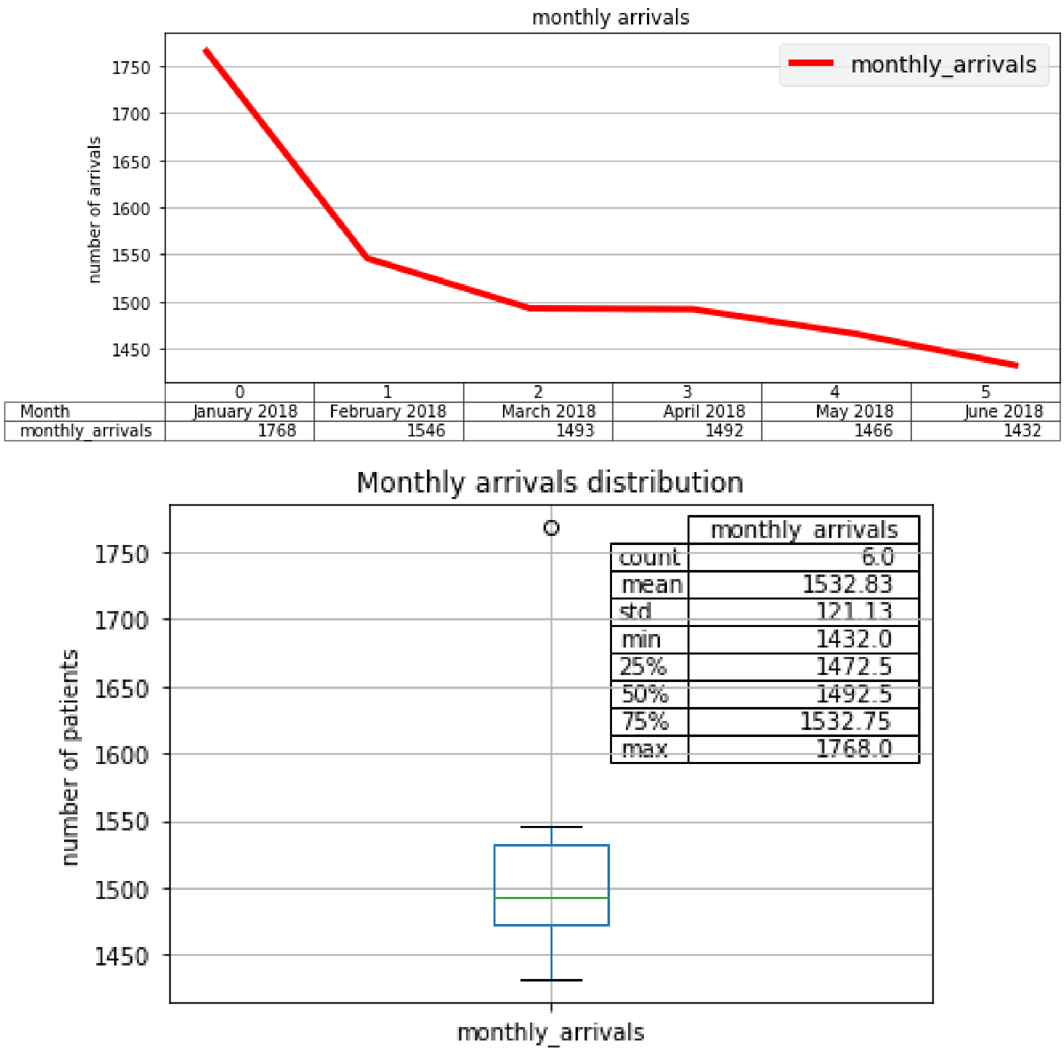
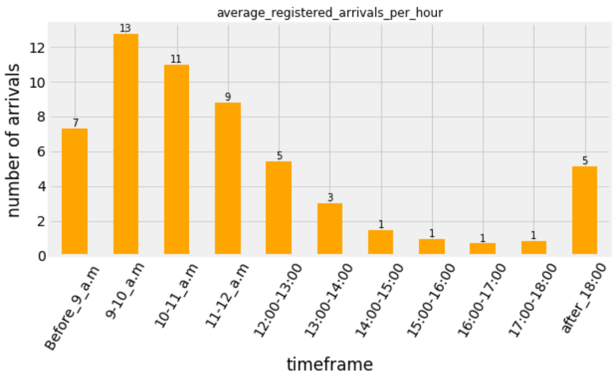


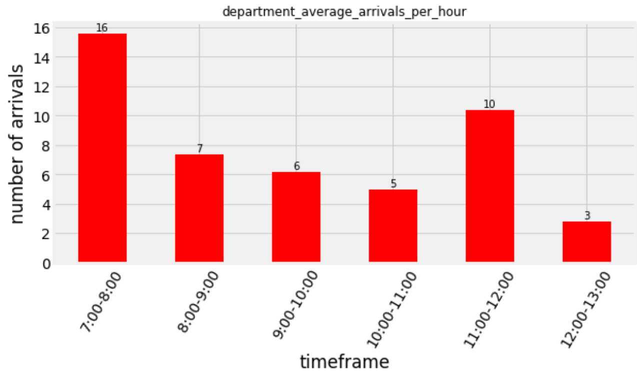

Figure 4.9: Monthly patient arrivals

By assumption, the number of arrivals could be reducing maybe because of the crisis or standards are lowering without being really noticed.

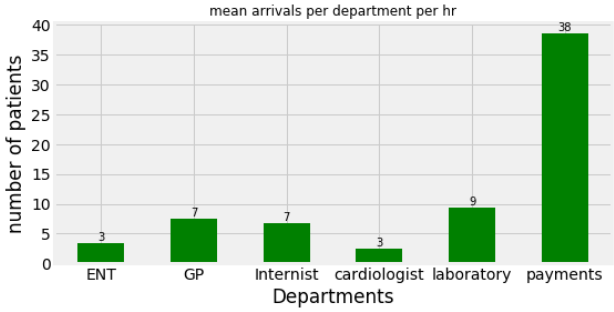
**4.2.2 Arrivals per hour.** The analysis had to be made as per movement of patients hourly. Figure 4.10a has the number of arrivals recorded in the OPD (screening room), excluding those coming for rendezvous, while Figure 4.10b shows the overall number of arrivals in the $6$ selected departments (service segments) for all patients including those coming for rendezvous.



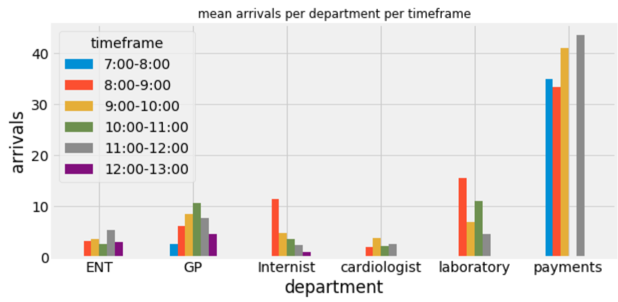

(a)




(b)




(c)




(d)

Figure 4.10: Hourly arrivals

Observing the trend, most patients arrive in the morning and the numbers decrease as the day goes by, although after hours, emergency cases tend to increase. Department wise, the payment section experiences the highest number of arrivals, most probably because it is the starting point. These patients are then distributed to the various consultation service segments. Among the specialists, the internist seems to be the most burdened.

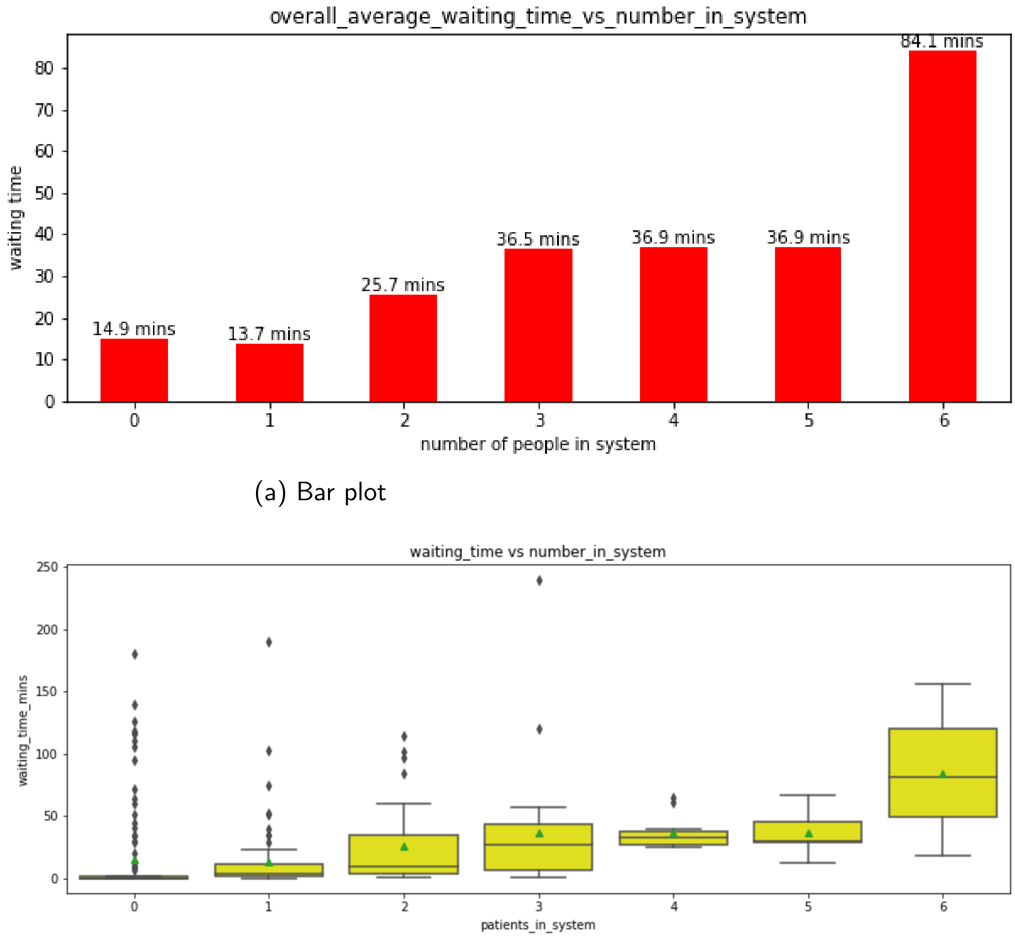
**4.2.3 How many people arrive in an hour and are served within the same hour of arrival?.** Figure 4.11 gives an overview on average of the variation of patients arriving and those being served in that same hour. From observations and results, it shows that on average most patients upon arrival are less likely to be served within that same hour of arrival. The larger the gap between arrivals and served, the more waiting time experienced. On the other hand, the correlation matrix, with $84\%$ correlation shows the existence of a relationship between the number of arriving patients vs the number served.


Figure 4.11: Number of arrivals vs number served

## 4.3 Observations: Time

All the observed time in this study is converted to minutes.

**4.3.1 If there are patients already in the system how long does a patient wait?.** Generally, when a patient arrives with no one already in the system, they are not supposed to have waiting time. Figure 4.12 and Figure 4.13 belong to the same observation set, except that the former is an overall observation of the departments while the latter is specific to each department. Both figures show that on average even if there are $0$ people in the system, a patient still has to wait in the queue and this factor alone triggers the waiting time as other incoming patients might also be affected. Figure 4.12a displays the overall average of waiting time vs patients already found in the system, whereas Figure 4.12b further shows the distribution and detail on the waiting times observed for each category.




(a) Bar plot

(b) Box plot

Figure 4.12: Waiting time vs $0$ to $\geq 6$ patients already in the system

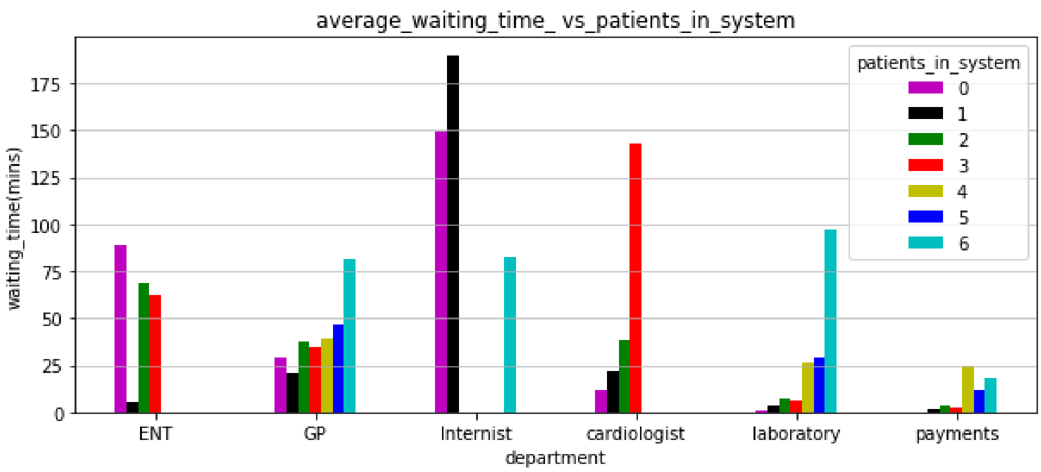



Figure 4.13: Department waiting time when patients are already in the system

In Figure 4.13, the missing bars of 'patients_in_system' for some service segments is as a result of no observations having been done. All the same, patients still wait for long even when they are the first ones in the queue.

**4.3.2 Waiting, service and total time.** Averages for waiting, service and total time spent in the hospital by patients in the selected service segments were calculated. Of all the services, the internist had the highest times compared to others.

For comparison and better understanding between days of the week and times spent in the hospital, time distributions were plotted, although only Tuesday to Friday was captured since observations could not happen on Mondays. Wednesday and Tuesday, in order, proved to have on average higher times compared to other days, where the averages are indicated in green.

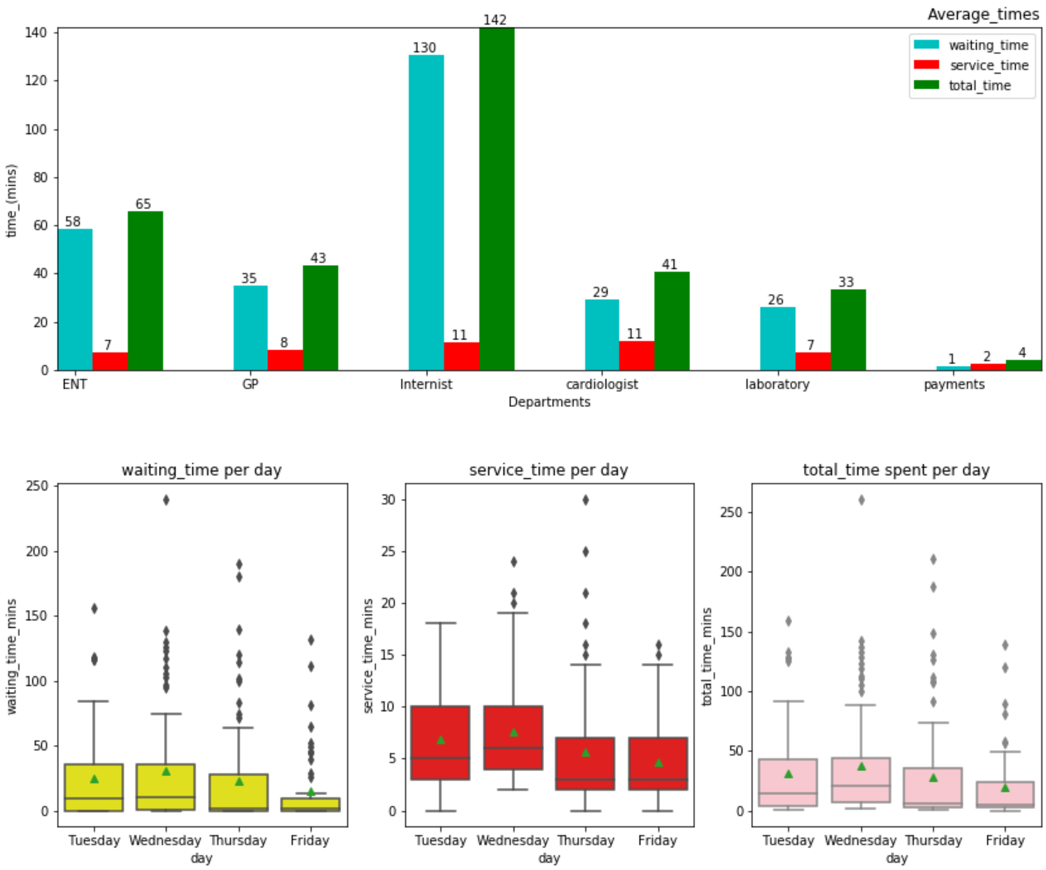



Figure 4.14: Observed times in the hospital

**4.3.3 Laboratory findings.** According to the information and calculations which were done by one of the laboratory personnels, average Turn Around Times (TATs) were calculated using a sample of $5$ patients for each laboratory test. TAT is defined as the total time it takes from when a sample is submitted to when the results come out. For the research, only $6$ common tests were selected and the results are shown in Table 4.2. From the results, no tests are below $56$ mins, probably explaining why patients are kept waiting for long.

Table 4.2: TAT for RHL laboratory results

| **Laboratory test** | **TAT (hrs)** |
|---|---|
| Mp (Malaria parasite) | $0:56$ |
| Hb (Haemoglobin) | $1:09$ |
| FBC (Full Blood Count) | $1:45$ |
| MS | $2:19$ |
| FBS (Glycaemia) | $2:34$ |
| Stool analysis | $3:50$ |

**4.3.4 Relationships between attributes.** For correlation clarity, Figure 4.15 was constructed to see if there exists a relationship among variables. Of all the variables, only waiting time and total time spent show a significant relationship with a strong positive correlation of $0.99$.

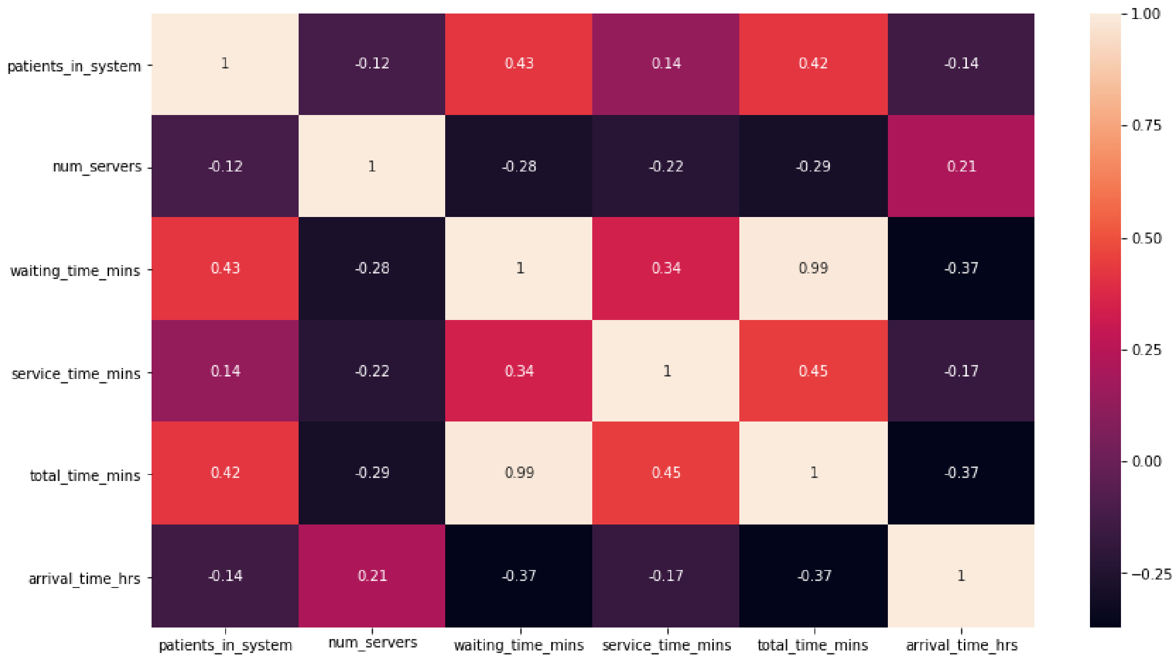



Figure 4.15: Correlation of variables

## 4.4 Machine Learning: Time predictions

Using random forests tree, predictions were made on both waiting and total time spent by a patient in the hospital tomorrow, where the output of the tree is random each time the notebook is run. Also, the variables important for predicting both parameters were calculated and constructed according to their importance.

**4.4.1 Waiting time predictions.** The Mean Absolute Error MAE for predicting waiting time was $10.53$, with a Median Absolute Error (MedAE) of $3.54$. The MAE value implies that the model predicts with an average of $10.53$ mins of error. To show how accurately the trained model fits the dataset, the

R-squared value was calculated and was found to be $0.64$, showing that the model prediction is almost close to the actual data set.

Figure 4.16 shows that to predict the waiting time, the most important variables are the time of arrival and the number of patients one will find in the system. The cardiologist, however, comes out as the $3^{rd}$ important variable, which is rather interesting as it is unexpected.

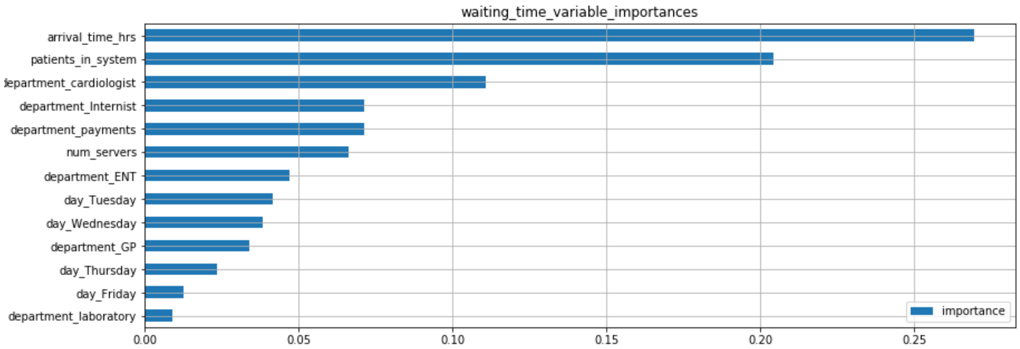



Figure 4.16: Variables important for predicting waiting time

In Figure 4.17, given the variables on the tree, a patient is thus able to predict how long they can wait in the hospital before receiving service.

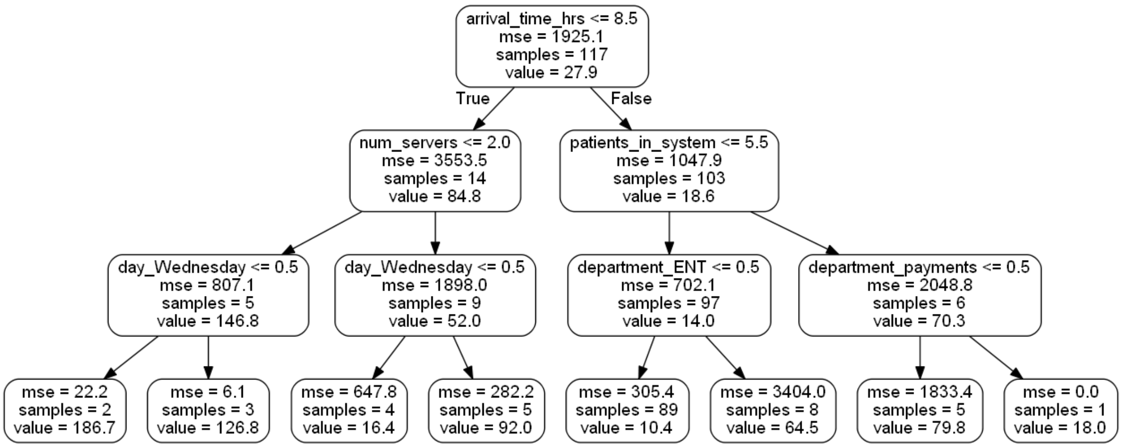



Figure 4.17: Waiting time predictions

To interpret this random forest in particular, from the first node, if a patient arrives before $8:50$ a.m, they choose the arrow indicated true, else if otherwise, they choose the false arrow. If the first node was true, the next question encountered is the number of servers. We move down the forest answering questions up to the last row whose nodes indicate the predicted waiting times which are given as 'value' in minutes. As an example, if a patient arrives before $8:50$ a.m and the service has less than $2$ servers on a Wednesday, the waiting time prediction is $186.7$ minutes.

The 'samples' shown on each node are indications of the number of samples randomly taken for sampling data points and 'MSE' is the Mean Squared Error.

**4.4.2 Total time predictions.** In reality, predicting total time spent by a patient is somewhat infeasible since the outcome cannot be predicted without a patient going through the process of waiting and receiving the service first. Therefore this prediction serves as a verifier of methods rather than giving prediction information beforehand. However, after training the model, the MAE was found to be $1.75$, implying that on average $1.75$ mins of error are predicted by the model. The MedAE was $0.77$ and the MAPE was $7.45\%$.

After obtaining Figure 4.18, the most important variables for total time prediction were waiting and service time. Again, the cardiologist service segment is the $3rd$ important variable, although in this case, it is not as much significant.

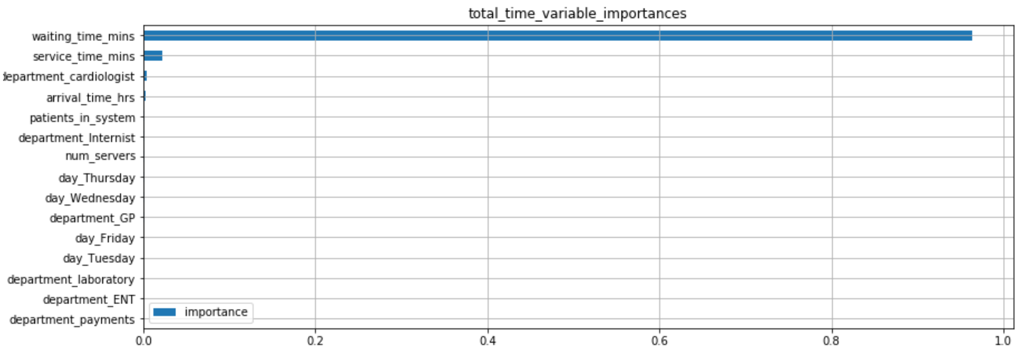



Figure 4.18: Variables important for predicting total time

Removing all other variables and remaining with the $2$ important ones, the MAE reduced to $1.4$, while the MedAE decreased to $0.39$ and the MAPE decreased to $5.23\%$. This implies all other features were not important for total time prediction after all. Scoring the model on training data, the R-squared value was $0.99$, showing that the model prediction is very close to the actual data set.

Figure 4.19 illustrates the tree for total time.

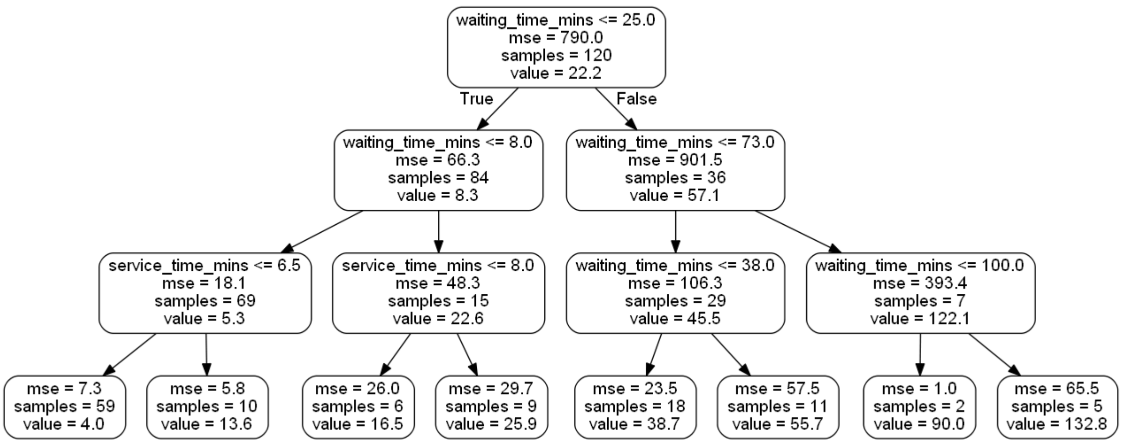



Figure 4.19: Total time predictions

**4.4.3 Error plots.** In order to see the samples from the validation dataset and the range within which their errors lie, error plots were constructed for both waiting and total time. It can be noted from the plots that most samples have small minute errors.

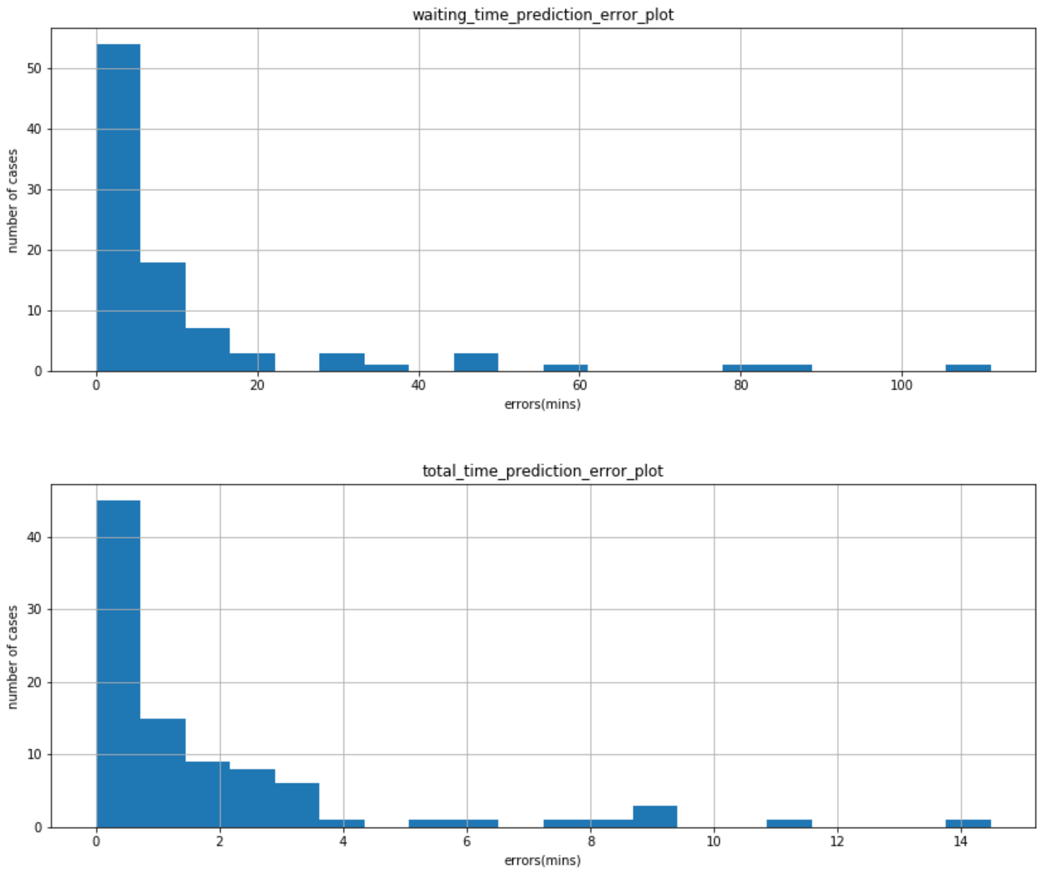



Figure 4.20: Prediction error plots

## 4.5 Queuing theory: Simulation results

After simulation of $1000$ replications, each of the $3$ doctors had a summary of results. Summary measures of performance for the simulations which were calculated include average time in the system, average time in the queue and percentage time server is idle. Time in the system is the total time a patient spends from arrival, waiting in the queue up to the time they receive service. Time in the queue is simply the waiting time before service. The time a server is idle is when a doctor is not busy.

We are $95\%$ confident that the true mean lies within the stated intervals. From Figure A.3 and Figure A.4, the red highlight indicates the waiting times for all the patients arriving first in the system. We note that for each first patient the waiting time experienced was mostly more than $30$ mins before they were attended to.

From the results of Figure 4.21 to Figure 4.26, 'queuing results with disruptions' is the observed system, where the doctors' arrival times are inconsistent and they engage in other work other than serving patients in the queue. On the other hand, 'queuing results with no disruptions' is the model being compared to if the operating strategy was different, with doctors serving patients only without having other commitments.

| | Average time in system (mins) | Average time in queue (mins) | % time server is idle | Sample Standard deviation for average time in system | Sample Standard deviation for average time in queue | 95% Confidence interval for time in system | | 95% Confidence interval for time in queue | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | 51.28 | 47.78 | -35.81 | 169.30 | 0.00 | 149.66 |
| MIN | 6.36 | 1.2 | 0% | | | | | | |
| AVERAGE | 66.74 | 54.10 | 22.74% | Doctor_1 queuing results with disruptions | | | | | |
| MAXIMUM | 285.36 | 262.24 | 79.38% | | | | | | |

Figure 4.21: Queuing model for Doctor_1 with disruptions

| | Average time in system (mins) | Average time in queue (mins) | % time server is idle | Sample Standard deviation for average time in system | Sample Standard deviation for average time in queue | 95% Confidence interval for time in system | | 95% Confidence interval for time in queue | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | 3.57 | 3.17 | 3.76 | 18.05 | 0.00 | 10.30 |
| MIN | 5.04 | 0.08 | 10.84% | | | | | | |
| AVERAGE | 10.90 | 3.97 | 55.47% | Doctor_1 queuing results with no disruptions | | | | | |
| MAXIMUM | 39.84 | 30.8 | 78.56% | | | | | | |

Figure 4.22: Queuing model for Doctor_1 with no disruptions

Comparing the 2 systems for each doctor, it can be noted that there is a massive difference especially in the times a patient spends both in the queue and in the system. If we take doctor 1 for example, a patient spends on average 54 mins queuing in Figure 4.21, whereas in Figure 4.22 they will only spend 4 mins because the doctor has no other duties other than consulting. The same explanation is applicable for the differences noted in Figure 4.23 to Figure 4.26.

Since random numbers were generated, the output was random as well, meaning running the spreadsheet again would give slightly different results from what is seen. It is to be noted however, that simulation is an approximate method and might not give exact answers.

| | Average time in system (mins) | Average time in queue (mins) | % time server is idle | Sample Standard deviation for average time in system | Sample Standard deviation for average time in queue | 95% Confidence interval for time in system | | 95% Confidence interval for time in queue | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | 29.52 | 28.33 | -4.23 | 113.84 | 0.00 | 98.76 |
| MIN | 12.44 | 3.12 | 0.00% | | | | | | |
| AVERAGE | 54.80 | 42.11 | 19.31% | Doctor_2 queuing results with disruptions | | | | | |
| MAXIMUM | 177.24 | 158.48 | 70.40% | | | | | | |

Figure 4.23: Queuing model for Doctor_2 with disruptions

| | Average time in system (mins) | Average time in queue (mins) | % time server is idle | Sample Standard deviation for average time in system | Sample Standard deviation for average time in queue | 95% Confidence interval for time in system | | 95% Confidence interval for time in queue | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | 1.53 | 1.16 | 6.34 | 12.46 | 0.00 | 3.95 |
| MIN | 6.2 | 0 | 34.29% | | | | | | |
| AVERAGE | 9.40 | 1.63 | 64.18% | Doctor_2 queuing results with no disruptions | | | | | |
| MAXIMUM | 20.2 | 12.28 | 79.36% | | | | | | |

Figure 4.24: Queuing model for Doctor_2 with no disruptions

| | Average time in system (mins) | Average time in queue (mins) | % time server is idle | Sample Standard deviation for average time in system | Sample Standard deviation for average time in queue | 95% Confidence interval for time in system | | 95% Confidence interval for time in queue | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | 57.56 | 54.03 | -25.65 | 204.61 | 0.00 | 180.48 |
| MIN | 10.8 | 3.68 | 0.00% | | | | | | |
| AVERAGE | 89.48 | 72.42 | 20.03% | Doctor_3 queuing results with disruptions | | | | | |
| MAXIMUM | 350.2 | 318.44 | 74.39% | | | | | | |

Figure 4.25: Queuing model for Doctor_3 with disruptions

| | Average time in system (mins) | Average time in queue (mins) | % time server is idle | Sample Standard deviation for average time in system | Sample Standard deviation for average time in queue | 95% Confidence interval for time in system | | 95% Confidence interval for time in queue | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | 4.10 | 3.58 | 4.95 | 21.34 | 0.00 | 11.88 |
| MIN | 6.92 | 0.28 | 19.85% | | | | | | |
| AVERAGE | 13.15 | 4.72 | 63.19% | Doctor_3 queuing results with no disruptions | | | | | |
| MAXIMUM | 51.16 | 40.52 | 84.49% | | | | | | |

Figure 4.26: Queuing model for Doctor_3 with no disruptions

CIs play a role in modelling and simulation as they are used in model validation. The wide CI outputted in Figure 4.21, Figure 4.23 and Figure 4.25 indicates the small sample size which was used. In our case, standard deviation ($\sigma$) tells us how the collected time is spread out from the average.

# 5. Conclusions and Recommendations

## 5.1 Conclusions

After analysis performed, we draw conclusions on the findings and give recommendations by providing possible solutions.

The results and findings of this research made use of registers when Mondays were normal days and therefore the study assumes inexistence of ghost town. However given the case that ghost town is happening and patient arrivals has declined on Mondays, then these findings are not applicable and are subject to change only in terms of the busy days and the number of arrivals.

**5.1.1 Research questions.** In the first chapter, we had a set of research questions and this section responds to each of them according to the findings and outcome. The first question was on the exact cause of the queues to end up with long waiting time. After observing and analysing, we found that patient waiting time is sensitive to doctors' arrival time and the time they spend on other activities. This can be supported by the waiting times noted for all patients who arrived and found $0$ people in system.

The primary and secondary data played a major role in an attempt to answer the research questions. Firstly, from the combined patient and medical responses, the top $3$ busy departments were OPD, laboratory and Imaging centre. As much as these results were obtained from perceptions of $196$ people, without further investigations we concluded it to be true. All other observations necessary for the research were then carried out in $2$ of these departments.

For the busy days, from questionnaire perception, the top $3$ days were found to be Monday, Wednesday and Tuesday, at the same time observations indicated Monday, Wednesday, Tuesday to have the highest number of patient arrivals. This combined is evidence enough to conclude that Monday, Wednesday and Tuesday are the top $3$ busy days of the week for the hospital. As stated in the literature review from other studies, we can also compare with Mardiah [13] who found Monday to be the busiest day of the week. Again, when analysing the times spent in the hospital, Wednesday and Tuesday had the highest number of minutes of time spent by patients. Here, Monday could not be observed because of the crisis, however from secondary data findings, we can assume that it has the highest total time spent in the hospital.

We also wanted to find out if there could be a correlation existing between waiting lines and either day or time of the week. From assessment, there is an implication that waiting lines and the time spent is related with the day of the week. In simple form, we could put it in an equation as follows,

$$\uparrow \text{ in arrivals} \implies \text{long queues=busy day} \implies \text{more time spent in the hospital.} \tag{5.1.1}$$

In terms of busy times of the day, most medical personnel responded to mornings and again this matched with the observations from Figure 4.10. It is more congested and busy in the morning.

**5.1.2 Predictions with Machine learning.** In response to the predictions done, using the models developed, a patient can predict the amount of time they can wait tomorrow in the hospital. Figure 4.18 compliments Figure 4.15 using the fact that the waiting time is the most important variable for total time prediction, and to support that, the two were portrayed to have a strong correlation. The significance of the cardiologist service segment popping among the important variables implies that it contributes much to the waiting time in the hospital.

**5.1.3 Queuing theory.** In queuing theory, since the data was not enough to draw strong scientific conclusions, we base our conclusions on data analysis including simulation and assumptions. This implies the value of the essay is not in actual conclusions but rather in the methodology used. Using what-if analysis, an insight can be gained of the models in understanding and implementing waiting time strategies. We can only explain what can be done to get a more reliable appreciation of the current situation vs different possible working policies which are indicated in the recommendation section.

## 5.2 Summary

Table 5.1: Summary

| **Research questions** | **Subjective findings** |
|---|---|
| Main cause of long waiting time | Doctors' arrival time and commitment to other activities. |
| Departments with long waiting times | OPD, Laboratory and Imaging centre. |
| Busy days | Monday, Wednesday and Tuesday. |
| Busy time | Morning. |

## 5.3 Recommendations

**5.3.1 OPD Department.** Using the GP consultation case study of queuing theory to improve performance, several situations on operational changes could be considered and implemented;

1. Change queuing system: Use of different queuing rules like adopting "*Type* $2$*: One queue-Multiple servers*" queuing of Figure 3.3 or any other combination, where a patient is assigned to the first free/available doctor.
2. Increase doctors per shift: The queuing system setup could be changed by having an increase of $1$ or $2$ doctors consulting especially on busy days highlighted to reduce the waiting time.
3. Morning meetings: Doctors on duty could either send a representative or the meetings could be shifted to take place at a less busy time (afternoons).
4. Ward rounds: Doctors on duty at OPD could be exempted from ward rounds while non-consulting doctors do rounds and attend emergency cases.
5. Classify services: A class of patients requiring shorter service times like those coming for medical certificates can be dedicated to a different server. Waiting an hour for a signature and stamp that takes $2$ mins is not ideal.
6. For specialists consultation, consultation by appointment could be introduced.

### Triage

The process from triage to screening room is more or less a repetition and there is need to either combine or cut off some unnecessary steps and procedures to reduce the waiting time.

**Almoner (Payment section)**

The system is not computerised and a lot of time is spent invoicing by hand. If the system is changed, much difference will be noted.

**5.3.2 Laboratory Department.** From the findings concerning the long waiting time in the laboratory, it is due to the fact that results take long to come out (and or handed to the patient) than the time it takes to collect samples. As a result, the solution can only lie within reviewing the way the results are grouped or timed. Results should be produced in two shifts, $11$ a.m and $2$ p.m.

**5.3.3 Imaging centre.** This was among the departments mentioned to have long waiting time. However, because of the complexity setup and sensitivity of the process, no observations could be done. We can not make any conclusions but to recommend for future work.

## 5.4 General remarks

Without considering any department in particular but generalising for the hospital, these points could be taken into consideration;

1. Staff allocation: In general for all service segments, since Monday was found to be the busiest day of the week and morning is busier than during the day, staff allocation can be more concentrated to suit the days and times.
2. Entertainment: Providing patients with educational health talks, books, magazines e.t.c can also help while they are in the queue to keep them occupied. This can also help avoid reneging and baulking, as most patients get impatient and bored and leave without receiving service.

For future research and use, the type of operational data needed as input to a queuing model can be introduced since it is often unavailable in the registers. This is necessary for reviewing the system performance as well.

# Appendix A. Some additional data

REGIONAL HOSPITAL LIMBE (RHL):
HMIS (Statistics Department)

SURVEY ON PATIENT WAITING TIME AND SATISFACTION IN THE HOSPITAL

---

**Taking part in this survey is voluntary**
YOUR ANSWERS WILL BE TREATED IN CONFIDENCE

**Please tick where appropriate**

① Age range: ◯ Below 12 ◯ $13-19$ ◯ $20-39$ ◯ 40 and above

② Sex: ◯ M ◯ F

③ Marital Status: ◯ Single ◯ Married ◯ Other (specify) ...........

④ How often do you come to this hospital?
◯ First time ◯ Daily ◯ Weekly ◯ Monthly ◯ Emergency cases only
◯ Not sure

⑤ From your experience in this hospital, which section exhibits the longest waiting time? (*choose one option*)
◯ OPD/Consultation ◯ Insurance ◯ Laboratory ◯ Imaging center (xrays)
◯ Pediatric ◯ Dental ◯ Eye unit ◯ Surgical ◯ Other (specify) ...........

⑥ Under normal circumstances, what time do you prefer coming to this hospital?
◯ Morning $(8-12)$ ◯ Afternoon $(1-5)$ ◯ Evening (6− midnight) ◯ Anytime

**Use response ⑤ to answer the following**

⑦ On arrival do you find a seat in the waiting area? ◯ Yes ◯ No ◯ Not always

⑧ How would you rate the queue (line) lengths? ◯ Short ◯ Average ◯ Long

⑨ If the queue (line) is too long what do you do?
◯ Just wait ◯ Jump the queue ◯ Leave ◯ Other (specify) ...........

⑩ On average how long do you take waiting in the queue (line) before seeing a medical personnel? ◯ less than 10 mins ◯ $10-20$ mins ◯ $20-30$ mins
◯ 30 mins - 1 hour ◯ more than 1 hour

⑪ Approximately what is the average time taken during a medical service?
◯ less than 10 mins ◯ $10-20$ mins ◯ $20-30$ mins ◯ 30 mins - 1 hour
◯ more than 1 hour

⑫ On a scale $0-10$, (where 0 indicates **no** satisfaction and 10 indicates **maximum** satisfaction), rate the overall satisfaction of the waiting time spent in the hospital .........

Leave any comments/suggestions ..........................................................................................

1

Figure A.1: English sample questionnaire - (Patients)

AIMS | African Institute for Mathematical Sciences CAMEROON

**REGIONAL HOSPITAL LIMBE (RHL):**
**HMIS (Statistics Department)**

**MEDICAL PERSONNEL QUEUE LENGTH SURVEY**

---

**Taking part in this survey is voluntary**
YOUR ANSWERS WILL BE TREATED IN CONFIDENCE

**Please tick where appropriate**

① Age range: ◯ Below 30 ◯ 30 and above

② Sex: ◯ M ◯ F

③ Marital Status: ◯ Single ◯ Married ◯ Other (specify) ...........

④ Area of specialisation
◯ Specialist ◯ GP ◯ Nurse ◯ Dentist ◯ Lab technician ◯ Pharmacist
◯ Other (specify) ...........

⑤ Which department do you work in?
◯ Insurance ◯ Statistics ◯ Laboratory ◯ Eye unit ◯ Maternity ◯ Dental
◯ Imaging centre ◯ Pharmaceutical ◯ Surgical ◯ Social service ◯ Specialist/Medical
◯ Other (specify) ...........

⑥ Years of experience in your field ...... years

⑦ Which are the busiest days?
□ Everyday □ Sun □ Mon □ Tue □ Wed □ Thur □ Fri □ Sat

⑧ When is it busiest the most?
□ Morning $(8-12)$ □ Afternoon $(1-5)$ □ Evening (6− midnight) □ Anytime

⑨ On average how long does it take to attend to a patient/client?
◯ less than 10 mins ◯ $10-20$ mins ◯ $20-30$ mins ◯ 30 mins - 1 hour
◯ more than 1 hour

⑩ According to your opinion, which department(s) has the longest queues (lines)?
(*you can choose more than one option*)
□ OPD/Consultation □ Laboratory □ Insurance □ Imaging center
□ Pediatric □ Dental □ Surgical □ Eye unit □ Other (specify) ...........

⑪ How would you rate the queue lengths? ◯ Short ◯ Average ◯ Long

Leave any comments/suggestions below
.........................................................................................................................................

∗∗ *thank you for your participation* ∗∗

1

Figure A.2: English sample questionnaire - (Medical personnel)

QUEUING THEORY MODEL: OBSERVED 19-09-18 Wed

**Doctor_1**

| Patient no | Inter-arrival time (Mins) | Clock Arrival time | Service time (Mins | Clock Time service begins | Waiting time in queue (Mins) | Clock Time service ends | Time patient spends in system (Mins) | Idle time of server (Mins) |
|---|---|---|---|---|---|---|---|---|
| 1 | | 0 | 9 | 110 | 110 | 119 | 119 | 110 |
| 2 | 16 | 16 | 10 | 119 | 103 | 129 | 113 | 0 |
| 3 | 16 | 32 | 3 | 129 | 97 | 132 | 100 | 0 |
| 4 | 44 | 76 | 12 | 132 | 56 | 144 | 68 | 0 |
| 5 | 7 | 83 | 7 | 144 | 61 | 151 | 68 | 0 |
| 6 | 21 | 104 | 11 | 151 | 47 | 162 | 58 | 0 |
| 7 | 13 | 117 | 5 | 162 | 45 | 167 | 50 | 0 |
| 8 | 17 | 134 | 9 | 167 | 33 | 176 | 42 | 0 |
| 9 | 16 | 150 | 6 | 176 | 26 | 182 | 32 | 0 |
| 10 | 12 | 162 | 6 | 183 | 21 | 189 | 27 | 1 |
| 11 | 22 | 184 | 19 | 189 | 5 | 208 | 24 | 0 |
| 12 | 0 | 184 | 5 | 208 | 24 | 213 | 29 | 0 |
| 13 | 0 | 184 | 3 | 230 | 46 | 233 | 49 | 17 |
| 14 | 1 | 185 | 2 | 233 | 48 | 235 | 50 | 0 |
| 15 | 15 | 200 | 4 | 235 | 35 | 239 | 39 | 0 |
| 16 | 4 | 204 | 3 | 285 | 81 | 288 | 84 | 46 |
| 17 | 74 | 278 | 9 | 291 | 13 | 300 | 22 | 3 |
| 18 | 6 | 284 | 5 | 300 | 16 | 305 | 21 | 0 |
| 19 | 0 | 284 | 4 | 305 | 21 | 309 | 25 | 0 |
| Totals | 284 | | 132 | | 888 | | 1020 | 177 |
| Average | 15.7778 | | 6.94737 | | 46.7368 | | 53.68421 | |
| % idle time | | | | | | | | 57.28% |

**Doctor_2**

| Patient no | Inter-arrival time (Mins) | Clock Arrival time | Service time (Mins | Clock Time service begins | Waiting time in queue (Mins) | Clock Time service ends | Time patient spends in system (Mins) | Idle time of server (Mins) |
|---|---|---|---|---|---|---|---|---|
| 1 | | 0 | 10 | 35 | 35 | 45 | 45 | 35 |
| 2 | 75 | 75 | 7 | 75 | 0 | 82 | 7 | 30 |
| 3 | 11 | 86 | 11 | 96 | 10 | 107 | 21 | 14 |
| 4 | 8 | 94 | 10 | 113 | 19 | 123 | 29 | 6 |
| 5 | 17 | 111 | 1 | 123 | 12 | 124 | 13 | 0 |
| 6 | 3 | 114 | 2 | 124 | 10 | 126 | 12 | 0 |
| 7 | 32 | 146 | 9 | 166 | 20 | 175 | 29 | 40 |
| 8 | 20 | 166 | 7 | 179 | 13 | 186 | 20 | 4 |
| 9 | 10 | 176 | 6 | 186 | 10 | 192 | 16 | 0 |
| 10 | 28 | 204 | 7 | 204 | 0 | 211 | 7 | 12 |
| 11 | 3 | 207 | 8 | 211 | 4 | 219 | 12 | 0 |
| 12 | 34 | 241 | 15 | 241 | 0 | 256 | 15 | 22 |
| Totals | 241 | | 93 | | 133 | | 226 | 163 |
| Average | 21.9091 | | 7.75 | | 11.0833 | | 18.83333 | |
| % idle time | | | | | | | | 63.67% |

**Doctor_3**

| Patient no | Inter-arrival time (Mins) | Clock Arrival time | Service time (Mins | Clock Time service begins | Waiting time in queue (Mins) | Clock Time service ends | Time patient spends in system (Mins) | Idle time of server (Mins) |
|---|---|---|---|---|---|---|---|---|
| 1 | | 0 | 5 | 106 | 106 | 111 | 111 | 106 |
| 2 | 110 | 110 | 14 | 111 | 1 | 125 | 15 | 0 |
| 3 | 10 | 120 | 3 | 147 | 27 | 150 | 30 | 22 |
| 4 | 24 | 144 | 7 | 150 | 6 | 157 | 13 | 0 |
| 5 | 5 | 149 | 4 | 157 | 8 | 161 | 12 | 0 |
| 6 | 11 | 160 | 9 | 170 | 10 | 179 | 19 | 9 |
| 7 | 26 | 186 | 6 | 188 | 2 | 194 | 8 | 9 |
| 8 | 3 | 189 | 6 | 194 | 5 | 200 | 11 | 0 |
| 9 | 1 | 190 | 3 | 200 | 10 | 203 | 13 | 0 |
| 10 | 4 | 194 | 14 | 203 | 9 | 217 | 23 | 0 |
| 11 | 8 | 202 | 9 | 230 | 28 | 239 | 37 | 13 |
| 12 | 16 | 218 | 9 | 239 | 21 | 248 | 30 | 0 |
| 13 | 66 | 284 | 20 | 304 | 20 | 324 | 40 | 56 |
| Totals | 284 | | 109 | | 253 | | 362 | 215 |
| Average | 23.6667 | | 8.38462 | | 19.4615 | | 27.84615 | |
| % idle time | | | | | | | | 66.36% |

Figure A.3: GP observed queuing analysis

**QUEUING THEORY MODEL: ATTEMPTED OBSERVATIONS (Tue 25-09-18)**

| Doctor_1 | | | | | | | | | Doctor_2 | | | | | | | | | Doctor_3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Patient no | Inter-arrival time (Mins) | Clock Arrival time | Service time (Mins | Clock Time service begins | Waiting time in queue (Mins) | Clock Time service ends | Time patient spends in system (Mins) | Idle time of server (Mins) | Patient no | Inter-arrival time (Mins) | Clock Arrival time | Service time (Mins | Clock Time service begins | Waiting time in queue (Mins) | Clock Time service ends | Time patient spends in system (Mins) | Idle time of server (Mins) | UNAVAILABLE/ DELAYED |
| 1 | | **0** | 3 | **137** | 137 | 140 | 140 | 137 | 1 | | **0** | 3 | **0** | 0 | 3 | 3 | 0 | |
| 2 | 29 | 29 | 13 | 140 | 111 | 153 | 124 | 0 | 2 | 1 | 1 | 16 | 3 | 2 | 19 | 18 | 0 | |
| | | | | | | | | | 3 | 7 | 8 | 15 | 19 | 11 | 34 | 26 | 0 | |

Figure A.4: GP attempted queuing analysis observations

# Acknowledgements

I am grateful to God for the opportunities given. I would like to thank AIMS for giving me the opportunity to do an internship. In turn, I would like to thank the Director of Regional Hospital, Dr Denis Nsame, together with my supervisors, Mr Ekukole Serge and Mr Lukong for the insight, views and letting me apply my skills in the hospital. To my academic supervisors, Prof Habiboulaye and Prof Giovanni, it would not have been possible without your guidance. Many thanks to my tutor Philomena Marfo for all the hard work, commitment and contributions she made. I am also grateful for the support and encouragement I received from my colleagues. To my family, whom I dedicate this work, I am thankful for the prayers.

# References

[1] Adeleke, R., Ogunwale, O., and Halid, O. (2009). Application of queuing theory to waiting time of out-patients in hospitals. *Pac J of Sci Tech*, 10(2):270–4.

[2] Association, B. C. M. et al. (2006). *Waiting Too Long: Reducing and Better Managing Wait Times in BC: a Policy Paper by the BCMA's Council on Health Economics & Policy*. British Columbia Medical Association.

[3] Bamgboye, E., Erinoso, H., and Ogunlesi, A. (1994). The waiting time at the children's emergency room, university college hospital, ibadan. *Nig J Pediatr*, 19:9–14.

[4] Cochran, J. K. and Roche, K. T. (2009). A multi-class queuing network analysis methodology for improving hospital emergency department performance. *Computers & Operations Research*, 36(5):1497–1512.

[5] Conrad, M. (2013). Patient waiting time and associated factors at the assessment center, general out-patient department mulago hospital uganda.

[6] Curtis, C., Liu, C., Bollerman, T. J., and Pianykh, O. S. (2018). Machine learning for predicting patient wait times and appointment delays. *Journal of the American College of Radiology*, 15(9):1310–1316.

[7] Dansky, K. H. and Miles, J. (1997). Patient satisfaction with ambulatory healthcare services: waiting time and filling time. *Journal of Healthcare Management*, 42(2):165.

[8] Fomundam, S. and Herrmann, J. W. (2007). A survey of queuing theory applications in healthcare. Technical report.

[9] Fouogue, J. T., Tchounzou, R., Fouelifack, F. Y., Fouedjio, J. H., Dohbit, J. S., Sando, Z., and Mboudou, E. T. (2017). Evaluation of patients' satisfaction after laparoscopic surgery in a tertiary hospital in cameroon (africa). *The Pan African medical journal*, 28.

[10] Hall, R. W. (2006). Patient flow-the new queueing theory for healthcare. *OR MS TODAY*, 33(3):36.

[11] Kendall, D. G. (1953). Stochastic processes occurring in the theory of queues and their analysis by the method of the imbedded markov chain. *The Annals of Mathematical Statistics*, pages 338–354.

[12] Kim, S.-C., Horowitz, I., Young, K. K., and Buckley, T. A. (1999). Analysis of capacity management of the intensive care unit in a hospital. *European Journal of Operational Research*, 115(1):36–46.

[13] Mardiah, F. P. and Basri, M. H. (2013). The analysis of appointment system to reduce outpatient waiting time at indonesia's public hospital. *Human Resource Management Research*, 3(1):27–33.

[14] McQuarrie, D. (1983). Hospitalization utilization levels. the application of queuing. theory to a controversial medical economic problem. *Minnesota medicine*, 66(11):679.

[15] O'malley, M. S., Fletcher, S. W., Fletcher, R. H., and Earp, J. A. (1983). Measuring patient waiting time in a practice setting: A comparison of methods. *The Journal of ambulatory care management*, 6(3):20–27.

[16] Ozcan, Y. A. (2005). *Quantitative methods in health care management: techniques and applications*, volume 4. John Wiley & Sons.

[17] Putri, A. H. M., Subekti, R., and Binatari, N. (2017). The completion of non-steady-state queue model on the queue system in dr. yap eye hospital yogyakarta. In *Journal of Physics: Conference Series*, volume 855, page 012034. IOP Publishing.

[18] R.B, W. (2014). Factors associated with patient waiting time at a medical outpatient clinic: A case study of university of nairobi health services. page 32.

[19] Stewart, G., Rosenberg, N., et al. (1994). Pediatric emergency department walk-outs. *Pediatric emergency care*, 10(2):76–78.

[20] Sundarapandian, V. (2009). *Probability, statistics and queuing theory*. PHI Learning Pvt. Ltd.

[21] Troccoli, N., Capoor, B., and Troute, M. Predicting cs106 office hours queuing times.

[22] Vasanawala, S. S. and Desser, T. S. (2005). Accommodation of requests for emergency us and ct: Applications of queueing theory to scheduling of urgent studies. *Radiology*, 235(1):244–249.